Explaining the Yang–Mills Mass Gap with Observer-Patch Repair Dynamics

A Support-Visible OPH Route to the Clay Problem

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Abstract

The Clay Yang–Mills problem asks for a nontrivial four-dimensional quantum Yang–Mills theory for every compact simple gauge group, with a positive mass gap. Observer-Patch Holography gives a specific route to both pieces on its sharpened support-visible compact-gauge branch. The compact-gauge reconstruction, local holonomy variables, four-dimensional scaling chart, reflection-positive zero-obstruction vacuum, and MaxEnt/local-Gibbs principle determine the Euclidean continuum form

$$S_E[A] = \frac{1}{4g^2}\int_{\mathbb{R}^4} \langle F_{\mu\nu}, F_{\mu\nu}\rangle\, d^4x, \qquad F = dA + A\wedge A,$$

with the corresponding gauge-quotient Euclidean transfer semigroup. The mass-gap proof is kept separate from that construction. At fixed cutoff, exact local repair on an active collar is the vacuum-preserving conditional expectation onto the repaired visible data. The ground-state transformed Euclidean transfer generator is therefore a finite sum

$$L_r^{\mathrm{rep}} = \sum_{C\in\mathcal{C}_r} c_C(I - E_C), \qquad c_C > 0.$$

Refinement-stable locality gives a uniform positive rate floor $c_* > 0$. A bounded-color collar cover and repair completeness imply $L_r^{\mathrm{rep}} \geq c_*(I - P_{0,r})$ at every regulator. The compact-gauge refinement ladder and support-visible Hilbert-space extraction pass the exact finite-stage intertwining to the continuum:

$$UHU^{-1} = L^{\mathrm{rep}}.$$

Consequently the continuum compact-gauge Hamiltonian has

$$\mathrm{Spec}(H)\cap(0, c_*) = \varnothing, \qquad \Delta_{\mathrm{YM}} = \Delta_{\mathrm{rep}} \geq c_* > 0.$$

The result is group-uniform, and the accounting is exact: the Yang–Mills gap is the repair gap. No fitted phenomenological parameter enters the argument. The proof uses compact-gauge quotient locality, exact repair, bounded-color active collars, refinement coherence, support-visible continuum extraction, and independence from special features of the realized Standard Model quotient. Relative to the Clay/Jaffe–Witten statement, the external audit point is whether the OPH support-visible compact-gauge continuum extraction is accepted as the required four-dimensional axiomatic Yang–Mills construction on the declared branch.

1 Claim Boundary

This paper proves the Yang–Mills mass-gap theorem on the sharpened support-visible exact Euclidean-consensus branch of OPH. The branch data are compact-gauge reconstruction, four-dimensional Euclidean scaling, reflection positivity, the ordinary zero-obstruction vacuum sector, exact local repair as conditional expectation, bounded-color active collar covers, repair completeness, and support-visible compact-gauge continuum extraction.

More precisely, the imported four-dimensional form theorem uses compact-gauge reconstruction, the four-dimensional scaling chart, the reflection-positive ordinary vacuum sector, the absence of additional gauge-invariant relevant dimension-four pure-gauge operators beyond the positive quadratic curvature invariant, and the support-visible cylinder extraction. The later mass-gap step adds exact local repair, bounded-color active collars, and repair completeness.

Acceptance as a Clay-admissible solution depends on accepting the upgraded OPH compact-gauge support-visible extraction and Euclidean Yang–Mills form theorems as the four-dimensional axiomatic Yang–Mills construction required by the Clay/Jaffe–Witten statement. The proof below then isolates the mass-gap step and identifies the gap exactly:

$$\Delta_{\mathrm{YM}} = \Delta_{\mathrm{rep}}.$$

The two-dimensional heat-kernel identity in the wider OPH stack remains a separate normalization and worldsheet-effective bridge; it is not the mass-gap proof.

2 Position in the OPH Paper Stack

This paper is a focused companion to the OPH paper stack. The broad reconstruction program is summarized in *Observers Are All You Need* [1]. The compact technical core is *Recovering Relativity and the Standard Model from Observer Overlap Consistency* [2], which carries the support-visible compact-gauge repair-gap theorem inside the compact paper itself. The particle branch is separate [3]. The finite repair and quotient-normal-form machinery comes from *Reality as a Consensus Protocol* [4]. The regulated screen, record, and edge heat-kernel architecture is developed in *Federated Echosahedral Screen Microphysics* [5].

The edge-sector theorem in the stack relates OPH heat-kernel weights to a two-dimensional Yang–Mills partition identity. That identity fixes normalization and the controlled worldsheet effective bridge. The four-dimensional mass-gap argument uses repair dynamics: exact local repair becomes a positive Euclidean relaxation generator, and the uniform repair gap is transported to the compact-gauge Hamiltonian.

The Clay-facing import is the compact paper's D7 Yang–Mills theorem surface: the compact-gauge reconstruction ladder, the four-dimensional Euclidean Yang–Mills form theorem, the coherent compact-gauge extraction proposition, the support-visible Osterwalder–Schrader reconstruction theorem, the realized nontriviality step, and the exact repair-gap theorem. This note isolates that branch theorem surface; it does not enlarge it.

3 The Clay Target

The Clay Mathematics Institute describes the Yang–Mills mass gap as the missing mathematical foundation behind the quantum theory used for nonabelian gauge forces [6]. Jaffe and Witten state the problem as follows: for every compact simple gauge group G, construct a nontrivial quantum Yang–Mills theory on $\mathbb{R}^4$ satisfying axiomatic properties at least as strong as the stated Wightman or Osterwalder–Schrader references and prove a mass gap $\Delta > 0$ [7, 8].

The OPH proof below addresses that target through a different variable. The starting data are finite observer patches, compact-gauge visible quotient data, and exact repair collars. The nonzero energy threshold is the cost of leaving the repair-fixed vacuum sector.

4 Standing Setup

Fix a compact simple gauge group G carried by an OPH compact-gauge zero-obstruction vacuum branch, realized at fixed cutoff by the declared compact-gauge patch-carrier architecture. The architecture supplies finite local Hilbert spaces, exact local constraints, patch and overlap algebras, overlap sector projectors, record layers, and local repair interfaces.

Let r range over a cofinal refinement family of finite regulators. For each r, let

$$(\mathcal{H}_r, \Omega_r, H_r), \qquad T_r(t) = e^{-tH_r},$$

be the physical Euclidean Hilbert space, vacuum, Hamiltonian, and transfer semigroup.

Let X_r be the support-visible compact-gauge quotient state space, let π_r be the vacuum stationary measure, and set

$$K_r := L^2(X_r, \pi_r).$$

Let $\mathcal{C}_r$ be the finite family of active repair collars. For each active collar $C \in \mathcal{C}_r$, let

$$\rho_C : X_r \to Y_C$$

be the complete repaired visible datum, and let

$$E_C : K_r \to K_r$$

be conditional expectation onto the ρ_C-measurable functions.

The proof stays on the ordinary or central zero-obstruction vacuum branch. The genuinely noncentral higher-gauge branch is a different fixed-cutoff sector in the OPH stack and is not used for the ordinary compact-simple G theorem below.

Notation	Meaning
X_r, π_r, K_r	support-visible quotient state space, vacuum stationary measure, and $L^2(X_r, \pi_r)$.
$\mathcal{C}_r$	finite active repair-collar family at regulator r.
ρ_C	complete repaired visible datum on collar C.

Notation	Meaning
E_C	π_r-preserving conditional expectation onto ρ_C-measurable functions.
L_r^{rep}	ground-state transformed Euclidean repair generator at cutoff r.
$P_{0,r}$	projection onto constants in K_r, corresponding to the physical vacuum.
U_r	finite-stage unitary from the physical Euclidean Hilbert space to the repair L^2 space.
$K, \mathcal{H}, U$	support-visible continuum repair Hilbert space, physical Hilbert space, and limiting unitary.
c_*	uniform active-collar repair-rate floor.

5 Imported 4D Euclidean Yang–Mills Form

Assumption 5.1 (Support-visible compact-gauge Yang–Mills branch)**.** *The compact-gauge branch used below satisfies the following branch-local conditions.*

(i) the ordinary or central zero-obstruction compact-gauge sector survives refinement with compact simple structure group G;

(ii) the support-visible quotient carries a four-dimensional Euclidean scaling chart;

(iii) the ordinary vacuum sector is reflection positive and has topological angle $\theta = 0$;

(iv) the local finite-constraint MaxEnt/Gibbs family is gauge-invariant, Euclidean local, rotation-invariant, and refinement-stable;

(v) no additional gauge-invariant relevant dimension-four pure-gauge operator remains on this branch besides the positive quadratic curvature invariant;

(vi) the support-visible compact-gauge cylinder family has the weak-$*$ / GNS continuum extraction stated in Proposition 9.1.

Theorem 5.2 (Imported OPH branch theorem: four-dimensional Euclidean Yang–Mills form)**.** *Under Assumption 5.1, the continuum gauge-sector Euclidean action is*

$$S_E[A] = \frac{1}{4g^2} \int_{\mathbb{R}^4} \langle F_{\mu\nu}, F_{\mu\nu} \rangle \, d^4x, \qquad F = dA + A \wedge A, \tag{YM}$$

with compact simple structure group G. Equivalently, the support-visible continuum transfer semigroup is the Euclidean Yang–Mills semigroup associated with the gauge-quotient cylinder measure

$$d\mu_{\mathrm{YM}}(A) = Z^{-1} e^{-S_E[A]} \, DA/G$$

in the OPH support-visible GNS representation.

Proof. This is the support-visible compact-gauge Yang–Mills form theorem of the compact OPH paper, where it appears as Theorem `thm:oph-4d-euclidean-yang-mills-form` [2]. The proof spine is recalled here because it fixes the target Hamiltonian for the mass-gap step.

The compact-gauge branch reconstructs a compact group G from the zero-obstruction transportable bosonic sector category and its faithful fiber functor [2]. At fixed cutoff, the declared compact-gauge patch-carrier presentation gives support-visible link holonomies and plaquette holonomies. In the refinement limit, the zero-obstruction gluing law makes infinitesimal rectangle holonomies multiplicative and path-local. Therefore there is a local connection A on the four-dimensional scaling chart, and the infinitesimal plaquette defect is

$$U_{\mu\nu}(\varepsilon, x) = \mathbf{1} + \varepsilon^2 F_{\mu\nu}(x) + O(\varepsilon^3), \qquad F = dA + A \wedge A.$$

The Euclideanized MaxEnt/local-Gibbs branch gives a local finite-range action density built from support-visible gauge-invariant collar data. Gauge quotienting permits only class functions of the curvature and its covariant derivatives. The four-dimensional scaling chart, Euclidean rotation invariance, locality, and reflection positivity leave one relevant dimension-four positive quadratic invariant in the pure gauge sector:

$$\langle F_{\mu\nu}, F_{\mu\nu} \rangle.$$

The possible topological density $\langle F \wedge F \rangle$ is reflection odd and belongs to a separate topological-angle sector; it is absent on the ordinary reflection-positive zero-obstruction vacuum branch used here. Higher curvature powers and covariant-derivative terms are irrelevant operators under the declared continuum scaling and vanish from the strict Yang–Mills fixed form. Normalizing the unique positive quadratic invariant defines the coupling g and gives (YM).

The finite-stage cylinder measures are the gauge-register / quantum-link Gibbs measures pushed to the support-visible quotient. Assumption 5.1 supplies the compatible support-visible weak-$*$ / GNS cylinder extraction. The resulting Euclidean transfer semigroup is therefore the transfer semigroup of (YM). □

Remark 5.3 (Why this step matters for the prize)*.* The proof has two separate claims. Theorem 5.2 identifies the support-visible continuum gauge sector with four-dimensional Euclidean Yang–Mills. The spectral argument applies to that Hamiltonian and proves a positive gap.

6 Fixed-Cutoff Repair Equals Projection

Proposition 6.1 (Local exact repair equals conditional expectation)**.** *For each active collar C, the exact-Markov repair map on the support-visible quotient is the π_r-preserving conditional expectation E_C.*

Proof. On the exact-Markov branch, repair preserves exactly the repaired visible datum ρ_C, changes only complementary invisible fiber data, and acts on the quotient-first physical algebra rather than on microscopic representatives. Let Φ_C be the Heisenberg repair map and let $\mathcal{N}_C$ be the repaired local fixed algebra. The repair semantics give

$$\Phi_C(a) = a \quad (a \in \mathcal{N}_C),$$

$$\Phi_C(\mathcal{A}_r^{\mathrm{sv}}) \subseteq \mathcal{N}_C, \qquad \pi_r \circ \Phi_C = \pi_r,$$

and Φ_C is $\mathcal{N}_C$-bimodular. Therefore, for $a \in \mathcal{N}_C$ and $x \in \mathcal{A}_r^{\mathrm{sv}}$,

$$\pi_r(a^* \Phi_C(x)) = \pi_r(a^* x).$$

Since $\Phi_C(x) \in \mathcal{N}_C$ and π_r is faithful, this equation uniquely characterizes $\Phi_C(x)$ as the orthogonal projection of x onto $\mathcal{N}_C$ in the GNS inner product. That orthogonal projection is the π_r-preserving conditional expectation E_C. □

7 Exact Euclidean-Consensus Law

Lemma 7.1 (Implementation hiding gives fiber permutation symmetry)**.** *Fix an active collar C and a repaired value $y \in Y_C$. On the support-visible quotient, the hidden fiber*

$$F_C(y) = \rho_C^{-1}(y)$$

has no remaining observable labels. Consequently the conditioned local MaxEnt state is uniform on $F_C(y)$, and the primitive collar relaxation commutes with all finite permutations of $F_C(y)$.

Proof. The quotient removes implementation labels by construction: two representatives with the same repaired datum y and different hidden fiber coordinates define the same support-visible observable state. The MaxEnt rule conditioned on y therefore has no admissible support-visible constraint that can distinguish two points of $F_C(y)$. The unique constraint-compatible conditioned state is the uniform state on that finite fiber. Any primitive collar relaxation preserving the OPH quotient must commute with the resulting full permutation action. □

Lemma 7.2 (Scalar relaxation on a uniform hidden fiber)**.** *Let F be a finite hidden fiber with uniform measure, let E_F be expectation onto constants, and let D_F be a positive self-adjoint Markov relaxation generator such that*

$$\ker D_F = \operatorname{Ran}(E_F)$$

and D_F commutes with the full permutation group of F. Then there is a scalar $c_F > 0$ such that

$$D_F = c_F(I - E_F).$$

Proof. The permutation representation on $L^2(F)$ splits as constants plus the zero-sum subspace. The zero-sum subspace is irreducible for the full symmetric group when $|F| \geq 2$. Schur's lemma therefore makes D_F a scalar on that subspace. Positivity and the kernel condition make the scalar strictly positive. The formula follows. □

Theorem 7.3 (Exact Euclidean repair law)**.** *There are positive constants $c_C > 0$ such that the ground-state transformed physical Euclidean generator is exactly*

$$L_r^{\mathrm{rep}} = \sum_{C \in \mathcal{C}_r} c_C(I - E_C), \tag{1}$$

and therefore

$$U_r e^{-tH_r} U_r^{-1} = e^{-tL_r^{\mathrm{rep}}} \qquad (t \geq 0), \tag{2}$$

for a unitary $U_r : \mathcal{H}_r \to K_r$ with $U_r \Omega_r = \mathbf{1}_r$.

Proof. The OPH MaxEnt axiom gives a local-Gibbs state with a quasi-local finite-range generator on the declared finite-constraint branch. After the support-visible quotient, each primitive local

Euclidean piece D_C is supported on one collar C, preserves exactly the repaired visible datum ρ_C, and relaxes only complementary fiber data. Hence

$$\ker D_C = \mathrm{Ran}(E_C).$$

Lemma 7.1 gives the full hidden-fiber permutation symmetry. Applying Lemma 7.2 fiberwise gives a positive scalar c_C on the orthogonal complement of the repaired datum:

$$D_C = c_C(I - E_C).$$

The scalar is positive because C is active. Branch homogeneity makes it a collar-type scalar, so summing over the active collars gives (1), and exponentiation of the positive self-adjoint generator gives (2). □

Lemma 7.4 (Uniform active-collar rate floor). *Assume finite local combinatorial type, a branch-homogeneous local constraint family, and refinement-stable exact repair semantics. Then the local Euclidean repair rate depends only on active collar type, the active collar-type set is finite across the cofinal refinement family, and there is $c_* > 0$, independent of r, such that*

$$c_C \geq c_* \qquad \textit{for every active collar } C \in \mathcal{C}_r. \tag{3}$$

Proof. Active collars are exactly the collars on which complementary invisible fiber data are genuinely relaxed. Hence $D_C \neq 0$ on $\ker(E_C)$, so $c_C > 0$. Finite local combinatorial type gives a finite list of bounded collar patterns, including their visible interface alphabets, hidden fiber cardinalities, local constraint templates, and admissible repair maps. Branch homogeneity makes the conditioned MaxEnt relaxation scalar a function of this finite type data. Refinement stability says that refinement replaces a collar by copies of the same bounded type templates with the same normalized local Euclidean repair semantics, so no new rate-degenerating collar type appears along the cofinal family. Taking the minimum of c_C over the finite active type list gives $c_* > 0$. □

8 Finite-Stage Gap

Proposition 8.1 (Finite-stage repair gap). *Assume that the collar family admits a bounded-color decomposition*

$$\mathcal{C}_r = \bigsqcup_{a=1}^{q} \mathcal{C}_{r,a}, \qquad q < \infty, \tag{4}$$

independent of r, and that exact quotient-local gluing makes the corresponding color expectations commute. Then

$$L_r^{\mathrm{rep}} \geq c_*(I - P_{0,r}), \tag{5}$$

where $P_{0,r}$ projects onto constants in K_r. Hence

$$\Delta_{\mathrm{rep},r} \geq c_* > 0. \tag{6}$$

Proof. For each color a, define the parallel color expectation

$$E_{r,a} := \prod_{C \in \mathcal{C}_{r,a}} E_C.$$

Same-color collars are disjoint, so the factors commute. Exact quotient-local gluing makes $\{E_{r,a}\}_{a=1}^{q}$ a commuting family of orthogonal projections on the vacuum branch.

Repair completeness says that the only support-visible observables fixed by every local repair are constants:

$$\bigcap_{a=1}^{q} \mathrm{Ran}(E_{r,a}) = \mathbb{C}\,\mathbf{1}_r. \tag{7}$$

Define

$$\widetilde{L}_r := \sum_{a=1}^{q} c_*(I - E_{r,a}).$$

For commuting projections,

$$I - \prod E_C \le \sum_C (I - E_C),$$

so $\widetilde{L}_r \le L_r^{\mathrm{rep}}$. The projections $E_{r,a}$ are simultaneously diagonalizable. On the joint all-ones eigenspace, (7) says the vector is constant. Every nonconstant joint eigenspace has at least one color eigenvalue zero, so

$$\widetilde{L}_r \ge c_*(I - P_{0,r}).$$

This proves (5), and the spectral gap bound (6) follows. □

9 Continuum Extraction

Proposition 9.1 (Coherent refinement and support-visible extraction)**.** *Under Assumption 5.1, the compact-gauge branch supplies the coherence and extraction data needed for the continuum intertwining theorem:*

(1) finite-stage support-visible quotient state spaces X_r, algebras $\mathcal{A}_r^{\mathrm{sv}}$, stationary states π_r, and Hilbert spaces $K_r = L^2(X_r, \pi_r)$;

(2) refinement maps $\jmath_{rs}^{\mathrm{rep}} : K_r \to K_s$ and $\jmath_{rs}^{\mathrm{phys}} : \mathcal{H}_r \to \mathcal{H}_s$ compatible with all repaired local marginals;

(3) a projectively compatible family of finite-stage compact-gauge cylinder marginals;

(4) weak-$*$ compactness and a diagonal subnet whose local marginals converge on every support-visible cylinder algebra;

(5) support-visible GNS gluing to continuum Hilbert spaces K and $\mathcal{H}$, with dense images of the local cylinder algebras;

(6) a faithful limiting vacuum pair on the support-visible gauge-invariant algebra after quotienting the maximal repair-invariant overlap-trivial kernel.

Proof. The compact-gauge ladder in the OPH compact paper proves fixed-cutoff bosonic sector categories, faithful monoidal refinement functors, compatible finite-dimensional fibers, the directed colimit sector category, and compact gauge reconstruction [2]. The support-visible compact-gauge quotient algebra is generated by overlap sector projectors and compact-gauge visible bosonic observables, modulo the maximal repair-invariant overlap-trivial kernel.

Each regulator has finite-dimensional state space and finite local cylinder algebras, so its state space is weak-$*$ compact. Refinement compatibility makes the local marginals projective: the restriction of a refined cylinder marginal to any coarser cylinder equals the coarser marginal. Choosing a countable cofinal family of support-visible cylinders and applying a diagonal subnet argument gives a limiting state on their algebraic union. Positivity and normalization pass to the limit on every cylinder, and the quotient by the repair-invariant overlap-trivial kernel makes the limit faithful on the support-visible gauge-invariant algebra.

The GNS construction applied to this limiting state gives K. Transporting the same coherent refinement data through the finite-stage intertwiners U_r gives the physical limiting Hilbert space $\mathcal{H}$. These are exactly the support-visible continuum objects used below. □

Theorem 9.2 (Continuum exact transfer identification)**.** *There is a unitary*

$$U : \mathcal{H} \to K$$

such that

$$Ue^{-tH}U^{-1} = e^{-tL^{\mathrm{rep}}} \qquad (t \geq 0), \tag{8}$$

and hence

$$UHU^{-1} = L^{\mathrm{rep}}. \tag{9}$$

Proof. Theorem 7.3 gives exact finite-stage intertwining. Proposition 9.1 supplies the direct-limit and support-visible GNS extraction hypotheses. Therefore the finite intertwiners pass to the continuum. Equality of generators follows from uniqueness of the self-adjoint generator of a strongly continuous contraction semigroup. □

10 Axiomatic Reconstruction and Nontriviality

Theorem 10.1 (Osterwalder–Schrader reconstruction on the support-visible compact-gauge branch)**.** *Under Assumption 5.1 and Proposition 9.1, the continuum support-visible compact-gauge cylinder family is Euclidean invariant, reflection positive, regular on gauge-invariant local cylinder observables, and cyclic for the vacuum sector. Hence Osterwalder–Schrader reconstruction gives a four-dimensional quantum Yang–Mills theory*

$$(\mathcal{H}, \Omega, H, \mathcal{A}^G_{\mathrm{loc}})$$

on the support-visible gauge-invariant local algebra, with $H \geq 0$ and e^{-tH} equal to the Euclidean transfer semigroup of Theorem 5.2.

Proof. Euclidean covariance is part of the four-dimensional scaling chart and the local-Gibbs cylinder family. Reflection positivity is part of the ordinary vacuum branch in Assumption 5.1. Regularity follows from the finite local cylinder construction and the support-visible weak-$*$ limit in Proposition 9.1. The vacuum vector is cyclic for the GNS closure of the gauge-invariant local cylinder algebra by construction. The standard Osterwalder–Schrader reconstruction theorem therefore produces the Hilbert space, vacuum, local algebra, and positive Hamiltonian, and identifies the physical time-translation semigroup with the Euclidean transfer semigroup [9, 10]. □

Proposition 10.2 (Nontriviality of the support-visible compact-gauge theory)**.** *The support-visible compact-gauge local algebra on the zero-obstruction vacuum branch strictly contains the vacuum scalars and admits a non-vacuum finite-energy local excitation.*

Proof. The compact-gauge witness and physical-UV landing theorem in the OPH compact paper supplies a realized nontrivial compact-gauge branch [2]. At finite cutoff this gives a support-visible gauge-invariant local observable, such as a nonconstant Wilson/plaquette cylinder observable, whose vacuum variance is positive. Its GNS vector is orthogonal to the vacuum after subtracting its expectation value. The finite-range local-Gibbs generator assigns finite energy to finite-cylinder excitations. Proposition 9.1 transports these local cylinder vectors into the support-visible continuum, so the continuum local algebra is strictly larger than $\mathbb{C}I$ and contains non-vacuum finite-energy local excitations. □

11 Main Theorem

Theorem 11.1 (Positive four-dimensional compact-gauge Yang–Mills mass gap)**.** *Let G be a compact simple gauge group carried by a support-visible compact-gauge OPH vacuum branch satisfying the standing setup above. The theory reconstructed in Theorem 10.1 is nontrivial by Proposition 10.2, and its continuum support-visible Hamiltonian H satisfies*

$$H \geq c_*(I - P_0), \tag{10}$$

where P_0 projects onto the vacuum. Therefore

$$\operatorname{Spec}(H) \cap (0, c_*) = \varnothing, \qquad \Delta_{\mathrm{YM}} \geq c_* > 0. \tag{11}$$

Moreover, the repair gap and Yang–Mills gap are exactly equal:

$$\Delta_{\mathrm{YM}} = \Delta_{\mathrm{rep}}. \tag{12}$$

Proof. At every finite stage, Proposition 8.1 gives

$$L_r^{\mathrm{rep}} \geq c_*(I - P_{0,r}).$$

By Proposition 9.1 and Theorem 9.2, this lower bound passes to the support-visible continuum:

$$L^{\mathrm{rep}} \geq c_*(I - P_0).$$

Using (9), $UHU^{-1} = L^{\mathrm{rep}}$. Conjugating the lower bound by U^{-1} gives (10), and the spectral statement (11) follows. Since unitary equivalence preserves the nonzero spectrum and identifies the vacuum with the constant sector, the infimum of the nonzero spectrum is the same on both sides, giving (12). □

Remark 11.2 (Group-uniform form)*.* The proof is group-uniform. Once a compact simple G is carried by a compact-gauge zero-obstruction OPH branch, no step uses special properties of the realized Standard Model quotient. The inputs are compact-gauge support-visible quotient locality, exact local repair on collars, bounded-color collar covers, refinement coherence, and support-visible continuum extraction.

12 Exact Gap Accounting

The proof gives more than a positive lower bound. It identifies the Hamiltonian whose gap is being measured:

$$H = U^{-1} L^{\mathrm{rep}} U.$$

Therefore

$$\mathrm{Spec}(H) \setminus \{0\} = \mathrm{Spec}(L^{\mathrm{rep}}) \setminus \{0\}.$$

The Yang–Mills gap is exactly the first nonzero repair eigenvalue:

$$\Delta_{\mathrm{YM}} := \inf\bigl(\mathrm{Spec}(H) \setminus \{0\}\bigr) = \inf\bigl(\mathrm{Spec}(L^{\mathrm{rep}}) \setminus \{0\}\bigr) =: \Delta_{\mathrm{rep}}.$$

The finite-stage color argument proves $\Delta_{\mathrm{rep}} \geq c_* > 0$. The exact accounting statement is the equality $\Delta_{\mathrm{YM}} = \Delta_{\mathrm{rep}}$; the inequality is the positivity proof for that same quantity.

13 Relation to the 2D Yang–Mills Bridge

The OPH corpus also contains an exact edge-sector identity:

$$Z_{\mathrm{edge}}(t) = \sum_R d_R^2 e^{-tC_2(R)} = K_t(1),$$

which is the compact-group heat kernel at the identity and matches the standard two-dimensional Yang–Mills heat-kernel partition form. Peter–Weyl supplies the heat-kernel identity, and Gross–Taylor gives the standard large-N worldsheet dictionary when a separate large-N_{edge} branch with remainder control is declared [11, 12].

That 2D result is a normalization and worldsheet-effective bridge. The spectral theorem above concerns the support-visible compact-gauge Hamiltonian and obtains its lower bound by identifying Euclidean transfer with the repair generator.

14 Clay Deliverables Checklist

Clay/Jaffe–Witten target	OPH repair-dynamics status
Compact simple gauge group G	G is arbitrary compact simple, provided it is carried by the OPH compact-gauge zero-obstruction branch.
Four-dimensional Euclidean Yang–Mills form	Theorem 5.2, imported from the compact paper's D7 Yang–Mills theorem surface, gives $S_E[A] = \frac{1}{4g^2} \int_{\mathbb{R}^4} \langle F_{\mu\nu}, F_{\mu\nu} \rangle \, d^4x$ under the branch assumptions in Assumption 5.1.

Clay/Jaffe–Witten target	OPH repair-dynamics status
Four-dimensional quantum Yang–Mills theory	Supplied through the support-visible compact-gauge continuum extraction of Proposition 9.1 and OS reconstruction in Theorem 10.1. The Clay-facing admissibility claim is exactly that this branch-local support-visible extraction supplies the required four-dimensional axiomatic construction.
Nontriviality	Proposition 10.2 gives a non-vacuum finite-energy local excitation in the support-visible compact-gauge algebra.
Axiomatic strength	Theorem 10.1 states the OS reconstruction step on the support-visible gauge-invariant local algebra.
Mass gap	The repair generator satisfies $L^{\mathrm{rep}} \geq c_*(I - P_0)$, and unitary transfer gives $H \geq c_*(I - P_0)$.
Exact gap accounting	The unitary identity $UHU^{-1} = L^{\mathrm{rep}}$ gives $\Delta_{\mathrm{YM}} = \Delta_{\mathrm{rep}}$. OPH accounts exactly for the Yang–Mills gap as the first nonzero repair eigenvalue.
Uniformity in G	The proof uses compact simplicity and compact-gauge quotient locality, not Standard-Model-specific representation data.

15 Conclusion

The OPH answer to the Yang–Mills mass gap is simple in mechanism. On the sharpened exact Euclidean-consensus branch, the gap statement is the operator identity $UHU^{-1} = L^{\mathrm{rep}}$, followed by spectral positivity. A non-vacuum support-visible compact-gauge excitation is not fixed by all local repair collars. At least one active collar must relax it. Because active collar rates have a uniform positive floor and the collars can be organized into finitely many commuting colors, every non-vacuum state pays a positive Euclidean repair cost. The continuum compact-gauge Hamiltonian is unitarily equivalent to that repair generator. Therefore the Yang–Mills gap is the repair gap.

$$\boxed{\Delta_{\mathrm{YM}} = \Delta_{\mathrm{rep}} \geq c_* > 0.}$$

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