# Observer-Patch Holography and the Dark Matter Phenomenon

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**Abstract**

The dark matter problem is easy to state and hard to make into one theory: galaxies, clusters, lensing maps, cosmic structure, and the cosmic microwave background all gravitate as if the observed baryons are an incomplete source. Particle dark matter gives a clean large-scale stress component. The particle is unobserved. MOND gets one striking galaxy fact right: in settled, low-acceleration disks the observed field behaves roughly as $g_{\rm obs} \simeq \sqrt{a_0 g_b}$, which gives the baryonic Tully-Fisher law. Its hard part is the static form of the rule: lensing, merging clusters, and the early universe need relativistic stress, transport, and perturbation equations.

In Observer-Patch Holography, gravity emerges from the consistency demand that finite observer patches agree on their shared overlap data. The ordinary Einstein branch is the continuum stress bookkeeping of stable overlap agreement. When agreement is imperfect, finite overlap collars carry a record-repair remainder. That electromagnetically dark remainder gravitates as an information-defect stress correction. In old, settled, low-acceleration galaxies this stress branch approximates the MOND acceleration law, because the repair opportunities sourced by baryonic acceleration have relaxed to a static collar equilibrium. In a settled one-dimensional galaxy the collar activation equation

$$\nabla^2\Phi = \nabla\cdot\left[\nu_\lambda\left(\frac{|\nabla\Phi_b|}{a_{0,\rm OPH}}\right)\nabla\Phi_b\right], \qquad \nu_\lambda(x) = \frac{1}{1-\exp[-\lambda_{\rm collar}\sqrt{x}]}.$$

has the MOND-like deep-galaxy limit $g_{\rm obs} \simeq \sqrt{a_{0,\rm eff}\, g_b}$. Away from that settled branch, OPH follows its stress-sector equations; the MOND interpolation rule is limited to the settled branch. This is why the usual MOND pressure points change form in OPH: on no-slip galaxy states the same potential controls dynamics and lensing; in cluster mergers the anomaly is transported with a finite repair time, so it can be displaced from the collisional gas; and in FLRW and perturbation theory it is handled as a homogeneous and linear stress component, with the static RAR excluded as a universal linear kernel. A protected quotient-center reserve gives a coefficient target within 1.748% of the common empirical acceleration reference. The SPARC disk-potential, cluster, and Boltzmann likelihood contracts are explicit; the real likelihood evaluations remain work in progress.

## Contents

# 1 Plain Picture

A galaxy does two jobs in this model. Its baryons source the ordinary weak field, and its finite observer-overlap collars carry a record-repair load. The load is dark to electromagnetic probes because ordinary luminous charges are absent. It gravitates because OPH puts carried collar remainders into the effective stress bookkeeping.

The RAR shape has a simple origin. The relevant repair opportunities live on cuts, so their broad count scales like $r_M/r = \sqrt{g_b/a_{0,\mathrm{OPH}}}$. Independent local opportunity counts give a Poisson zero-count law. The active fraction is $1 - \exp[-\lambda_{\mathrm{collar}}\sqrt{g_b/a_{0,\mathrm{OPH}}}]$. The conservative field equation promotes that scalar response into a potential equation. The simple algebraic acceleration law is exact only when symmetry removes the curl term.

# 2 Three Claim Layers

The first layer is the static galaxy law. It contains the RAR shape, the BTFR limit, the effective dark density, the no-slip lensing rule, and the $\mathbb{Z}_6$/Poisson coefficient.

The second layer is the dynamic dark component. It treats the anomaly as a transported stress that relaxes toward the static equilibrium over $\tau_{\mathrm{rec}}$. Cluster offsets use this layer.

The third layer is cosmological dark matter. It uses the same stress in linear perturbation theory, with a background abundance, a relaxation rate, and an equilibrium kernel. CMB, BAO, lensing, growth, and $S_8$ test this layer.

# 3 The Dark Matter Challenge

In Newtonian gravity and in the weak-field limit of general relativity, a test particle in a circular orbit satisfies

$$\frac{v^2(r)}{r} = g(r) = \frac{GM(<r)}{r^2}. \tag{1}$$

If almost all gravitating matter is visible baryonic matter, then outside the bright disk the enclosed mass should stop growing quickly and the rotation speed should fall as $v(r) \propto r^{-1/2}$. Disk galaxies do not behave that way. Their outer rotation curves often remain flat, so the inferred dynamical mass grows approximately linearly with radius:

$$M_{\rm dyn}(< r) = \frac{v^2 r}{G}. \tag{2}$$

The galaxy problem is only one part of the evidence. Clusters need more gravity than the observed gas and galaxies provide. Weak lensing maps in merging systems such as the Bullet Cluster show gravitational potential peaks separated from the X-ray gas [12]. The CMB and large-scale structure prefer a gravitating component that is mostly non-baryonic, close to pressureless, and present before galaxies settle [5]. Planck's base ΛCDM parameter set gives $\Omega_c h^2 = 0.120 \pm 0.001$ and $\Omega_b h^2 = 0.0224 \pm 0.0001$, so most matter is not baryonic in that model [5].

The challenge includes more than "flat rotation curves". A good theory has to explain:

- galaxy rotation curves and the baryonic Tully-Fisher relation;
- the tight radial acceleration relation, abbreviated RAR;
- galaxy and cluster lensing;
- cluster offsets in mergers;
- CMB acoustic peaks, BAO, matter power, weak lensing, and $S_8$.

# 4 Standard Routes

## 4.1 Cold Dark Matter

The standard route adds a new non-baryonic matter component. In its simplest form it is cold, effectively collisionless, and pressureless on cosmological scales. ΛCDM uses the same component for CMB peaks, large-scale structure, clusters, lensing, and galaxy halos.

The particle identity is absent. Direct detection has produced no confirmed dark matter particle. Galaxy data show a tight connection between baryonic acceleration and observed acceleration, much tighter than a naive halo-plus-disk picture suggests. Halo fitting also introduces galaxy-level nuisance freedom, while the RAR looks like a law.

## 4.2 MOND

MOND should be treated as a clue, not as a failure. Its basic phenomenology starts from a critical acceleration $a_0$, and in the deep low-acceleration regime it gives

$$g \simeq \sqrt{a_0 g_b}. \tag{3}$$

For a point baryonic mass this implies

$$v^4 = G M_b a_0, \tag{4}$$

which is exactly the baryonic Tully-Fisher scaling [10]. This is MOND's great success: it goes straight at the galaxy regularity.

The pressure points start when the same static acceleration rule is asked to do more jobs than a galaxy equilibrium law can naturally do:

- *Relativistic completion and lensing.* A nonrelativistic force law does not by itself say which metric photons follow, or how the modified field contributes to the stress side of Einstein's equations. Relativistic completions such as TeVeS exist [11], but the lensing sector then depends on extra fields and closure assumptions.
- *Clusters.* MOND reduces the cluster mass discrepancy but does not remove the need for additional gravitating stress in clusters. Merging systems are especially sharp: if the anomaly is an instantaneous function of the local baryonic acceleration, the lensing peak wants to remain too tightly tied to the visible baryonic source.
- *The early universe.* The CMB acoustic peaks, BAO, matter growth, CMB lensing, and weak-lensing amplitudes need a homogeneous and perturbative gravitating component before settled galaxies exist. A static RAR law does not by itself define that background abundance or its linear response.
- *Non-spherical and time-dependent systems.* The algebraic relation $g \simeq \sqrt{a_0 g_b}$ is clean in high-symmetry systems. In real three-dimensional fields, curl terms, environmental dependence, transport, and relaxation can matter.

OPH does not treat the static RAR formula as the master law. It approximates MOND in the regime where the OPH stress sector has relaxed onto the old, settled, low-acceleration galaxy branch. It need not approximate MOND in mergers, clusters, or the early universe, because those systems are not static one-dimensional galaxy equilibria. The underlying object is an effective stress sector carried by finite overlap repair. In old relaxed disks it projects to a baryon-linked acceleration law. In lensing it enters the metric-potential bookkeeping. In cluster mergers it is transported and relaxes over a finite repair time. In FLRW and perturbation theory it has to be specified as a homogeneous charge, load current, and response kernel.

This is the sense in which the MOND pressure points dissolve in OPH. OPH treats those failures as wrong-regime extrapolations, without making the same algebraic law more complicated. The static galaxy formula is allowed to be a very good equilibrium approximation, while lensing, clusters, and cosmology are handled by the corresponding OPH stress, transport, and perturbation equations. Those equations must pass their own falsifiers. The claim is structural separation, with no immunity from data.

# 5 The OPH Route

Observer-Patch Holography starts from finite observer patches and the demand that neighboring patches agree on overlap data. The relevant OPH inputs for the dark branch are:

- finite overlap collars and fixed-cutoff collar algebras;
- edge and cut registers on those collars;
- read, compare, repair, commit, and refresh operations;
- a finite carried modular remainder on the collar;
- the recovered Einstein branch with a screen-capacity de Sitter scale.

The dark-sector proposal is:

The dark source is a transported modular/collar information-defect remainder. It gravitates because it contributes to the effective stress bookkeeping. It is electromagnetically dark because it is a record-repair remainder, not an ordinary luminous matter species.

No new particle species is postulated. The effective dark stress is the gravitating part of finite-observer information repair.

$$G_{ab} + \Lambda g_{ab} = 8\pi G \left( T_{ab}^{\mathrm{SM}} + T_{ab}^{\mathcal{I}} + T_{ab}^{\mathrm{repair}} \right). \tag{5}$$

The symbol $A$ in formulas below denotes this anomaly or information-defect component. It is an effective stress sector, with no literal particle species claim.

This differs from particle dark matter and MOND in a useful way. It is stress-first like dark matter, so lensing and cosmology have a natural slot. It is baryon-linked like MOND only on the settled galaxy branch, because that equilibrium response is activated by collar/cut repair opportunities sourced by baryonic acceleration. The important contrast with MOND is not the square-root galaxy law; OPH intentionally reproduces that approximation where the equilibrium assumptions apply. The contrast is where the law lives. In MOND, the galaxy formula is often treated as the modification itself. In OPH it is the equilibrium projection of a transported stress component. That is why lensing, clusters, and cosmology can have their own equations without giving up the galaxy regularity.

# 6 Static Galaxy Derivation

## 6.1 Source From The Carried Collar Remainder

On the support-visible BW branch the primary theorem is the cap automorphism identity. In the special type-I or effective local representation where the geometric generator may be written inside the cap algebra, the OPH modular decomposition has the local form

$$K_C = 2\pi B_C + K_C^{(\mathrm{anom})} + \mathrm{const.} \tag{6}$$

At finite regulator stage, the nonadditive modular part is carried on the overlap collar. The dark branch packages the galaxy-scale positive remainder as

$$R_C := I(A : D|B) \geq 0, \tag{7}$$

where $I(A : D|B)$ is conditional mutual information. This is a phenomenological carried-defect variable on the continuation branch: small CMI supplies a recovered comparison state and a fixed-collar replacement modulus, not a one-shot exact Markov normal form or an exact finite-cutoff Einstein source. The corresponding effective rest-energy density is written

$$\rho_A c^2 = \frac{15}{8\pi^2 \ell^4} R_C. \tag{8}$$

The weak-field Poisson equation becomes

$$\nabla^2 \Phi = 4\pi G(\rho_b + \rho_A). \tag{9}$$

**Conditional theorem 1, attractive sign.** If the carried galaxy remainder is $R_C = I(A : D|B)$ and the coarse-grained domain measure is nonnegative, then the anomaly is attractive in the Newtonian weak-field equation.

**Proof.** Conditional mutual information is nonnegative, so $R_C \geq 0$. Equation (8) gives $\rho_A \geq 0$. Equation (9) then shows that the anomaly enters with the same attractive sign as baryonic mass. □

The source identification is a theorem target.

### 6.2 The Acceleration Variable

Define the baryonic acceleration ratio

$$x := \frac{g_b}{a_{0,\mathrm{OPH}}}. \tag{10}$$

For a point baryonic source,

$$g_b(r) = \frac{GM_b}{r^2}, \qquad r_M := \sqrt{\frac{GM_b}{a_{0,\mathrm{OPH}}}}, \tag{11}$$

so

$$x = \left(\frac{r_M}{r}\right)^2, \qquad \sqrt{x} = \frac{r_M}{r}. \tag{12}$$

The OPH support assumption is that settled-galaxy repair opportunities live on codimension-one collar/cut support. A screen-area count would scale as $x$. A cut-count scales as $\sqrt{x}$, as in (12).

### 6.3 Poisson Activation

Let the mean number of independent local repair opportunities be

$$\mu(x) = \lambda_{\mathrm{collar}}\sqrt{x}. \tag{13}$$

The Poisson law does not follow from fixing this mean alone. Fixed mean alone would select a geometric distribution on nonnegative integers. The OPH claim needs the stronger collar-count premise: independent increments on disjoint collar intervals, rare local repair events, and refinement stability. Partition an interval $I$ with mean $\mu(I)$ into $m$ equal subintervals. If each subinterval has one activation opportunity with probability $\mu(I)/m + O(m^{-2})$, and the probability of two or more events in one subinterval is $O(m^{-2})$, then for every fixed $n$,

$$\Pr[N(I) = n] = \lim_{m\to\infty} \binom{m}{n} \left(\frac{\mu(I)}{m}\right)^n \left(1 - \frac{\mu(I)}{m}\right)^{m-n} = e^{-\mu(I)}\frac{\mu(I)^n}{n!}. \tag{14}$$

In particular, the zero-count probability is

$$\Pr[N(I) = 0] = \lim_{m\to\infty} \left(1 - \frac{\mu(I)}{m}\right)^m = e^{-\mu(I)}. \tag{15}$$

The probability that at least one repair opportunity is active is

$$p(x) = 1 - \Pr[N = 0] = 1 - \exp[-\lambda_{\mathrm{collar}}\sqrt{x}]. \tag{16}$$

The activation count is only the counting part of the static law. The source closure says that active scalar repair opportunities recover baryonic flux in the settled one-dimensional branch:

$$g_b = p(x) g_{\mathrm{obs}}. \tag{17}$$

Thus

$$g_{\mathrm{obs}} = \nu_{\mathrm{OPH}}(x) g_b, \qquad \nu_{\mathrm{OPH}}(x) = \frac{1}{1 - \exp[-\lambda_{\mathrm{collar}}\sqrt{x}]}. \tag{18}$$

**Conditional theorem 2, activation law.** Assume codimension-one collar/cut support, independent increments on disjoint collar intervals, rare local repair events, refinement stability, and the settled scalar flux-recovery closure (17). Then the settled one-dimensional galaxy response is (18).

**Proof.** Codimension-one support gives a mean count proportional to $\sqrt{x}$. The scalar coefficient is $\lambda_{\text{collar}}$. Independent increments and rare-event refinement give the Poisson law (14). The active probability is the complement of the zero-count event, which is (16). Solving $g_b = p(x)g_{\text{obs}}$ in one-dimensional symmetry gives (18). □

### 6.4 Conservative Potential Equation

For general disks and three-dimensional baryonic profiles, the algebraic vector rule

$$\mathbf{g}_{\text{alg}} = \nu_{\text{OPH}}(|\mathbf{g}_b|/a_{0,\text{OPH}})\mathbf{g}_b \tag{19}$$

need not be curl-free because

$$\nabla \times \mathbf{g}_{\text{alg}} = \nabla\nu_{\text{OPH}} \times \mathbf{g}_b. \tag{20}$$

A gravitational acceleration must be derived from a potential. The primary static OPH equation is therefore

$$\nabla^2\Phi = \nabla \cdot \left[\nu_{\text{OPH}}\left(\frac{|\nabla\Phi_b|}{a_{0,\text{OPH}}}\right)\nabla\Phi_b\right], \qquad \mathbf{g}_{\text{obs}} = -\nabla\Phi, \tag{21}$$

with

$$\nabla^2\Phi_b = 4\pi G\rho_b. \tag{22}$$

In one-dimensional symmetry, (21) integrates to

$$g_{\text{obs}} = \nu_{\text{OPH}}(g_b/a_{0,\text{OPH}})g_b + C. \tag{23}$$

The boundary condition $g_{\text{obs}} \to 0$ when the enclosed baryonic flux vanishes sets $C = 0$, so (18) is exact for spherical, planar, or one-dimensional symmetry. The SPARC tables in this note use the algebraic diagnostic; a full disk-potential solver is required for publication-grade rotation and lensing claims.

### 6.5 Newtonian And Deep-IR Limits

For $x \gg 1$, $p(x) \to 1$, hence

$$g_{\text{obs}} \to g_b. \tag{24}$$

For $x \ll 1$,

$$p(x) = \lambda_{\text{collar}}\sqrt{x} + O(x), \tag{25}$$

so

$$g_{\text{obs}} \simeq \frac{g_b}{\lambda_{\text{collar}}\sqrt{g_b/a_{0,\text{OPH}}}} = \sqrt{\frac{a_{0,\text{OPH}}}{\lambda^2_{\text{collar}}}g_b} = \sqrt{a_{0,\text{eff}}\,g_b}, \tag{26}$$

where

$$a_{0,\text{eff}} = \frac{a_{0,\text{OPH}}}{\lambda^2_{\text{collar}}}. \tag{27}$$

For a point source,

$$v^4 = GM_b a_{0,\text{eff}}. \tag{28}$$

**Theorem 3, BTFR.** The OPH activation law implies the baryonic Tully-Fisher scaling in the deep-IR limit.

**Proof.** For a circular orbit, $v^2/r = g_{\text{obs}}$. In the deep-IR limit, $g_{\text{obs}} = \sqrt{a_{0,\text{eff}} G M_b / r^2}$. Multiplying by $r$ gives $v^2 = \sqrt{G M_b a_{0,\text{eff}}}$, hence (28). □

### 6.6 Effective Dark Density

The anomaly can be represented as an equilibrium effective density:

$$\mathbf{g}_A = (\nu_{\text{OPH}} - 1)\mathbf{g}_b, \qquad \rho_{A,\text{eq}} = -\frac{1}{4\pi G}\nabla \cdot [(\nu_{\text{OPH}} - 1)\mathbf{g}_b]. \tag{29}$$

For a spherical system,

$$M_{\text{dyn}}(r) = \nu_{\text{OPH}}(x) M_b(r), \qquad M_A(r) = [\nu_{\text{OPH}}(x) - 1] M_b(r). \tag{30}$$

In the deep-IR point-source limit, $M_{\text{dyn}}(r) \propto r$, which is the effective mass profile behind flat rotation curves.

For a point baryonic source, the exact equilibrium expression is

$$M_A(r) = \frac{M_b}{\exp(\lambda_{\text{collar}} r_M / r) - 1}, \tag{31}$$

and

$$\rho_A(r) = \frac{M_b \lambda_{\text{collar}} r_M \exp(\lambda_{\text{collar}} r_M / r)}{4\pi r^4 [\exp(\lambda_{\text{collar}} r_M / r) - 1]^2}. \tag{32}$$

For an extended spherical baryonic profile, positivity of the settled effective density is the monotonicity condition

$$\frac{\mathrm{d}}{\mathrm{d}r}\left([\nu_{\text{OPH}}(x(r)) - 1] M_b(r)\right) \geq 0. \tag{33}$$

Profiles that violate this condition need finite pressure, shear, or transport terms instead of the settled dust equilibrium alone.

## 7 The Finite-Collar Coefficient

### 7.1 Unit Branch

The default branch is

$$\lambda_{\text{collar}} = 1, \qquad a_{0,\text{eff}} = a_{0,\text{OPH}}. \tag{34}$$

This branch sets the finite-collar opportunity density to one.

### 7.2 $\mathbb{Z}_6$ Reserve Branch

The OPH gauge structure contains the realized quotient

$$G_{\text{phys}} = \frac{SU(3) \times SU(2) \times U(1)}{\mathbb{Z}_6}. \tag{35}$$

The protected shared-edge reserve has mean

$$\epsilon_{\mathbb{Z}_6} = \frac{\bar{\ell}_{\text{shared}}}{|\mathbb{Z}_6|} = \frac{P/4}{6} = \frac{P}{24}. \tag{36}$$

Using

$$P = 1.630968209403959, \tag{37}$$

gives

$$\epsilon_{\mathbb{Z}_6} = 0.06795700872516496. \tag{38}$$

**Conditional bridge 4, quotient-edge co-registration.** Galaxy scalar repair opportunities are quotient-edge cut-register events on the same edge-center resolution that carries the protected $\mathbb{Z}_6$ reserve. The $\mathbb{Z}_6$ reserve therefore co-registers with the scalar activation count.

**Conditional argument.** The scalar source is a quotient-local recoverability defect, so pure $\mathbb{Z}_6$ center motion changes no realized matter or Higgs state and is inactive for scalar sourcing. If the scalar count and the protected reserve are forced onto the same center-projector resolution, the reserve removes active scalar opportunities from that same layer. □

**Proof obligation.** The fixed-cutoff screen-net construction must prove that galaxy scalar activation cannot live in a separate bulk, screen-area, or noncentral collar channel.

**Conditional bridge 5, local Poisson reserve survival.** If the reserve occupancy in a co-registered local scalar slot at transverse collar coordinate $y$ is Poisson with mean $\epsilon_{\mathbb{Z}_6}(y)$, then the local scalar survival factor is

$$\lambda_{\text{slot}}(y) = \Pr[N_{\mathbb{Z}_6}(y) = 0] = \exp[-\epsilon_{\mathbb{Z}_6}(y)]. \tag{39}$$

**Proof.** A Poisson variable with mean $\epsilon_{\mathbb{Z}_6}$ has zero-count probability $\exp(-\epsilon_{\mathbb{Z}_6})$. A scalar opportunity survives only when its co-registered reserve count is zero. This proves the local survival factor. □

### 7.3 Finite Thickness

Let $y$ be the transverse collar coordinate and $w(y)$ the normalized activation weight:

$$\int \mathrm{d}y\, w(y) = 1. \tag{40}$$

The finite-thickness branch gives

$$\lambda_{\text{collar}} = \int \mathrm{d}y\, w(y) \exp[-\epsilon_{\mathbb{Z}_6}(y)]. \tag{41}$$

If

$$\int \mathrm{d}y\, w(y)\epsilon_{\mathbb{Z}_6}(y) = P/24, \tag{42}$$

then Jensen's inequality gives

$$\lambda_{\text{collar}} \geq \exp(-P/24). \tag{43}$$

Equality holds when $\epsilon_{\mathbb{Z}_6}(y)$ is constant on the weighted collar.

**Conditional bridge 6, exact finite-thickness coefficient.** On the product transverse regulator with one quotient trace, the exact finite-thickness coefficient is $\lambda_{\text{collar}} = \exp(-P/24)$.

**Conditional argument.** If the finite-thickness reserve density is uniform on the scalar-weighted collar layer, then $\epsilon_{\mathbb{Z}_6}(y) = \epsilon_{\mathbb{Z}_6}$. Equation (41) becomes

$$\lambda_{\text{collar}} = \exp(-\epsilon_{\mathbb{Z}_6}) \int \mathrm{d}y\, w(y) = \exp(-\epsilon_{\mathbb{Z}_6}). \tag{44}$$

Using $\epsilon_{\mathbb{Z}_6} = P/24$ proves the claim. □

On that exact branch,

$$\lambda_{\text{collar}} = \exp(-P/24) = 0.934300639489386\dots \tag{45}$$

and by (27)

$$a_{0,\text{eff}} = 1.179018696 \times 10^{-10}\,\mathrm{m\,s^{-2}}. \tag{46}$$

**Failure consequence.** Without finite-thickness uniformity, the coefficient is the integral (41). Exact $\exp(-P/24)$ is not available as a theorem-grade number.

# 8 Lensing And Relativistic Stress

A density source by itself fixes the Newtonian Poisson equation. Lensing also depends on the spatial stress. The OPH no-slip claim uses the transported record-number stress and its spatial moments.

The conditional OPH stress closure is a kinetic record measure. At each coarse-grained event, the finite collar packet emits a positive measure $\mathrm{d}\mathcal{E}_x(v)$ on the future unit timelike shell. Define

$$J^a_{\mathcal{I}}(x) = \int v^a\, \mathrm{d}\mathcal{E}_x(v), \qquad T^{ab}_{\mathcal{I}}(x) = \int v^a v^b\, \mathrm{d}\mathcal{E}_x(v). \tag{47}$$

This stress is symmetric and future-positive. Relative to an observer $u^a$, it decomposes as

$$T^{ab}_{\mathcal{I}} = \rho_{\mathcal{I}} u^a u^b + P_{\mathcal{I}} h^{ab} + q^{(a}_{\mathcal{I}} u^{b)} + \pi^{ab}_{\mathcal{I}}. \tag{48}$$

The settled galaxy branch is the monokinetic rest branch $\mathrm{d}\mathcal{E}_x(v) = \rho_{\mathcal{I}}(x)\delta(v - u_{\mathcal{I}})\mathrm{d}\Omega$. Then

$$T^{ab}_{\mathcal{I}} = \rho_{\mathcal{I}} u^a_{\mathcal{I}} u^b_{\mathcal{I}}, \qquad P_{\mathcal{I}} = 0, \qquad q^a_{\mathcal{I}} = 0, \qquad \pi^{ab}_{\mathcal{I}} = 0. \tag{49}$$

In Newtonian gauge,

$$\mathrm{d}s^2 = a^2[-(1+2\Psi)\mathrm{d}\eta^2 + (1-2\Phi)\mathrm{d}x^2]. \tag{50}$$

The off-diagonal spatial Einstein equation gives

$$k^2(\Phi - \Psi) = 12\pi G a^2 \sum_i (\rho_i + P_i)\sigma_i. \tag{51}$$

The $\mathcal{I}$-sector contribution to the right-hand side vanishes on (49). If baryonic, neutrino, radiation, and repair-sector anisotropic stresses are absent or handled separately, then

$$\Phi = \Psi. \tag{52}$$

The lensing potential is therefore the same potential that controls dynamics:

$$\Phi_{\text{lens}} = \frac{\Phi + \Psi}{2} = \Phi. \tag{53}$$

This is where OPH behaves differently from a force-only MOND law. The correction is attached to stress bookkeeping, so the same potential controls slow matter and lensing on settled no-slip galaxy states. Cluster mergers and cosmological perturbations use the full decomposition (48); a velocity-spread branch can carry pressure, heat flux, or anisotropic stress.

## 9 Clusters And Transport

The static galaxy law is an equilibrium law. Merging clusters need a transported information-defect state. In the cold transported branch, the kinetic stress (47) reduces to a density and velocity field:

$$\partial_t \rho_{\mathcal{I}} + \nabla \cdot (\rho_{\mathcal{I}} \mathbf{v}_{\mathcal{I}}) = -\frac{\rho_{\mathcal{I}} - \rho_{\mathcal{I},\mathrm{eq}}[\rho_b]}{\tau_{\mathrm{rec}}}, \tag{54}$$

$$\partial_t \mathbf{v}_{\mathcal{I}} + (\mathbf{v}_{\mathcal{I}} \cdot \nabla)\mathbf{v}_{\mathcal{I}} = -\nabla\Phi, \tag{55}$$

$$\nabla^2\Phi = 4\pi G(\rho_b + \rho_{\mathcal{I}}). \tag{56}$$

Velocity-spread branches use the moment equations of (48) instead of (54).

The repair-commit theorem gives the form of the relaxation term. Once the finite collar packet algebra is declared, the fixed-cutoff packet state space, proposal rule, equilibrium weights, reversible transition matrix, and spectral gap are determined by packet microphysics. Let $\gamma_{\mathrm{rec}}$ be that dimensionless gap and $t_{\mathrm{commit}}$ the physical commit time. Then

$$\Gamma_{\mathrm{rec}} = \frac{\gamma_{\mathrm{rec}}}{t_{\mathrm{commit}}}, \qquad \tau_{\mathrm{rec}} = \frac{t_{\mathrm{commit}}}{\gamma_{\mathrm{rec}}}. \tag{57}$$

Bianchi consistency requires a repair-exchange stress:

$$\nabla_a (T_b^{ab} + T_{\mathcal{I}}^{ab} + T_R^{ab} + \cdots) = 0. \tag{58}$$

In quasi-covariant notation,

$$\nabla_a T_{\mathcal{I}}^{ab} = -\Gamma_{\mathrm{rec}}(\rho_{\mathcal{I}} - \rho_{\mathcal{I},\mathrm{eq}})u_{\mathcal{I}}^b, \tag{59}$$

and $T_R^{ab}$ carries the opposite exchange flux.

An OPH-side no-fit clock candidate is the Jacobi mismatch rate. Let $E_{ij} = C_{0i0j}$ be the electric Weyl tidal tensor measured in the local free-fall frame, equivalently the trace-free Newtonian Hessian in the weak field. The homogeneous FLRW Ricci focusing term is subtracted, because it does not create a local overlap-visible cluster offset. The scalar rate

$$\Gamma_J = \left(\frac{E_{ij}E^{ij}}{6}\right)^{1/4}, \qquad \tau_J = \Gamma_J^{-1} \tag{60}$$

is invariant under local rotations and has the right units. For a Newtonian point source,

$$E_{ij}E^{ij} = 6\left(\frac{GM}{r^3}\right)^2, \qquad \tau_J = \sqrt{\frac{r^3}{GM}}. \tag{61}$$

The numerical scale is:

| Scale | $M/M_\odot$ | $r$ in kpc | $\tau_J$ in Gyr |
|---|---|---|---|
| Inner spiral disk | $6.0 \times 10^{10}$ | 10 | 0.0608673 |
| Outer spiral disk | $6.0 \times 10^{10}$ | 30 | 0.316276 |
| Dwarf scale | $1.0 \times 10^{9}$ | 5 | 0.166692 |
| Cluster core | $1.0 \times 10^{14}$ | 300 | 0.244986 |
| Massive cluster aperture | $1.0 \times 10^{15}$ | 1000 | 0.471476 |

For $d_0 = 200\,\text{kpc}$ and $t = 0.2\,\text{Gyr}$, the massive-cluster aperture row gives $d(t) = 130.859\,\text{kpc}$.

The conditional physical-time conversion is

$$\Gamma_{\text{rec}}(x,t) = \Gamma_J(x,t), \qquad t_{\text{commit}}(x,t) = \frac{\gamma_{\text{rec}}(x,t)}{\Gamma_J(x,t)}. \tag{62}$$

The static RAR law does not determine this clock. Any positive relaxation rate leaves the same equilibrium density. The Jacobi clock requires the premise that repair timing is set by the overlap-visible tidal mismatch rate of transported information-defect records. The theorem-grade packet clock must derive this premise without using cluster offsets as selectors.

For a one-mode merger offset,

$$d(t) = d_0 \exp(-t/\tau_{\text{rec}}), \qquad \tau_{\text{rec}} = \frac{t}{\ln(d_0/d(t))}. \tag{63}$$

With $d_0 = 200\,\text{kpc}$ and $t = 0.2\,\text{Gyr}$:

| Retained offset | Required $\tau_{\text{rec}}$ |
|---|---|
| 50 kpc | 0.144 Gyr |
| 100 kpc | 0.288 Gyr |
| 134 kpc | 0.500 Gyr |
| 150 kpc | 0.696 Gyr |
| 164 kpc | 1.000 Gyr |

Settled galaxies impose

$$\tau_{\text{rec}} < \frac{T_{\text{set}}}{\ln(1/\epsilon)} \tag{64}$$

if the allowed tracking error after settling time $T_{\text{set}}$ is $\epsilon$. For $T_{\text{set}} = 5\,\text{Gyr}$, the bounds are 2.17 Gyr, 1.67 Gyr, and 1.09 Gyr for 10%, 5%, and 1% tracking error.

## 10 CMB, BAO, Growth, And $S_8$

The linear cosmology contract has two distinct branches. A conserved homogeneous information-defect branch obeys

$$\rho_A' + 3\mathcal{H}\rho_A = 0, \qquad \rho_A(a) = \rho_{A0}a^{-3}, \tag{65}$$

where primes denote conformal time and $\mathcal{H} = a'/a$. This branch behaves like a cold stress component if its kinetic measure is monokinetic and its homogeneous charge $Q_A = a^3\rho_A V_{\text{com}}$ is selected by OPH screen data.

A repair-exchange branch instead relaxes toward a finite-collar equilibrium source:

$$\rho_A' + 3\mathcal{H}\rho_A = -a\Gamma_{\text{rec}}(\rho_A - \rho_{A,\text{eq}}). \tag{66}$$

The factor $a\Gamma_{\text{rec}}$ is the conformal-time rate associated with the physical repair rate. Define

$$q_A = \frac{\rho_{A,\text{eq}}}{\rho_A}, \qquad \delta_{A,\text{eq}}(k,a) = B_A(k,a)\delta_b(k,a). \tag{67}$$

For a pressureless no-slip repair branch with energy exchange parallel to the anomaly four-velocity, the Newtonian-gauge perturbations reduce to

$$\delta_A' = -\theta_A + 3\Phi' - a\Gamma_{\text{rec}}q_A(\delta_A - \delta_{A,\text{eq}}), \tag{68}$$

$$\theta_A' = -\mathcal{H}\theta_A + k^2\Psi. \tag{69}$$

The no-slip statement here concerns the information-defect sector. The total metric slip depends on baryons, radiation, neutrinos, repair stress, and any anisotropic stress in a velocity-spread branch.

The finite scalar-load repair diagnostic gives

$$\gamma_{\rm rec} = 0.050717471, \qquad \tau_{\rm rec}/t_{\rm commit} = 19.717071.$$

It maps a cluster-scale relaxation window to the following commit times:

| $\tau_{\rm rec}$ | $t_{\rm commit}$ |
|---|---|
| 0.3 Gyr | 15.2152 Myr |
| 0.5 Gyr | 25.3587 Myr |
| 1.0 Gyr | 50.7175 Myr |

For the same relaxation window, a simple background timing diagnostic gives:

| $\tau_{\rm rec}$ | $z = 0$ | $z = 10$ | $z = 1100$ |
|---|---|---|---|
| 0.3 Gyr | 48.4092 | 2.36213 | 0.00205644 |
| 0.5 Gyr | 29.0455 | 1.41728 | 0.00123387 |
| 1.0 Gyr | 14.5228 | 0.708639 | 0.000616933 |

For a hybrid branch, repair relaxation is tiny during acoustic physics and large in low-Hubble settled environments. A cosmology claim requires a Boltzmann implementation and likelihood tests against CMB TT/TE/EE, CMB lensing, BAO, weak lensing, redshift-space distortions, and $S_8$.

# 11 Abundance And Kernel Targets

The static galaxy law fixes a settled response around resolved baryonic inhomogeneity. A cosmological dark-matter branch needs two extra OPH outputs:

$$\rho_A(a), \qquad B_A(k,a). \tag{70}$$

The first is the homogeneous anomaly abundance. The second is the linear equilibrium-source kernel in

$$\delta_{A,\rm eq}(k,a) = B_A(k,a)\delta_b(k,a). \tag{71}$$

## 11.1 Static FLRW Boundary

The nonlinear galaxy law cannot be inserted directly into FLRW perturbation theory. In an exactly homogeneous background there is no preferred baryonic acceleration vector. If one tries to Taylor-expand the static RAR law at $\mathbf{g}_b = 0$, the susceptibility is singular because $\nu_{\rm OPH}(x) \sim (\lambda_{\rm collar}\sqrt{x})^{-1}$. The static RAR branch therefore does not select a homogeneous anomaly density or a universal CMB kernel.

This is a useful boundary. It prevents a hidden fit from being smuggled into the galaxy law. The CMB branch has to declare its background anomaly charge and its perturbation kernel from OPH state selection or finite-collar microphysics.

### 11.2 Flat Capacity-Saturated State Selection

One clean homogeneous branch is flat capacity saturation. The de Sitter screen capacity fixes $H_{\mathrm{dS}}$, and the flat Friedmann constraint at $a = 1$ gives

$$\Omega_{A0} = 1 - \Omega_{\Lambda,\mathrm{OPH}} - \Omega_{b0} - \Omega_{\nu 0} - \Omega_{r0}, \qquad \Omega_{\Lambda,\mathrm{OPH}} = \left(\frac{H_{\mathrm{dS}}}{H_0}\right)^2. \tag{72}$$

The transported homogeneous dust branch is

$$\rho_A(a) = \rho_{A0} a^{-3}. \tag{73}$$

Using $H_0 = 67.4\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$, $\Omega_b h^2 = 0.02237$, the OPH neutrino mass sum, and $\Omega_r = 9.17\times10^{-5}$, the residual is:

| Quantity | Value |
|---|---|
| $H_{\mathrm{dS}}$ | $55.759940256\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ |
| $\Omega_{\Lambda,\mathrm{OPH}}$ | 0.684423332 |
| $\Omega_b$ | 0.049243191 |
| $\Omega_\nu$ | 0.002127375 |
| $\Omega_r$ | 0.000091700 |
| $\Omega_A$ | 0.264114401 |
| $\Omega_m$ | 0.315484968 |
| $\rho_{A0}$ | $2.253649933 \times 10^{-27}\,\mathrm{kg\,m^{-3}}$ |

This matches the Planck-like cold-matter budget at the compressed-parameter level. The number is a state-selection residual. The static galaxy RAR law does not determine it. The OPH theorem that would make it standalone is a finite-screen additive-load selector for the conserved homogeneous anomaly charge.

The closed transported branch does give a theorem-grade current and charge. Let $u_A^a$ be the branch four-velocity and let $\rho_A$ be the coarse-grained information-defect load density. The load current is

$$J_A^a = \rho_A u_A^a. \tag{74}$$

On the closed branch,

$$\nabla_a J_A^a = 0, \qquad Q_A[\Sigma] = \int_\Sigma J_A^a n_a \,\mathrm{d}\Sigma \tag{75}$$

is independent of the homogeneous slice. In FLRW this gives

$$Q_A = a^3 \rho_A V_{\mathrm{com}}, \qquad \rho_A(a) = \frac{Q_A}{a^3 V_{\mathrm{com}}}. \tag{76}$$

This proves transport of a supplied homogeneous load. It does not fix the amplitude $Q_A$. The missing OPH object is a nontrivial finite-screen additive load invariant

$$Q_A = F_{\mathrm{OPH}}(N_{\mathrm{scr}}, P, \text{screen normal-form data})\,. \tag{77}$$

Without that invariant, the abundance is cosmological state data or the flat capacity-saturated residual (72).

### 11.3 Environmental Kernel

Around a nonzero local background field $\bar{\mathbf{g}}_b$, write

$$x = \frac{|\bar{\mathbf{g}}_b|}{a_{0,\mathrm{OPH}}}, \qquad \mathbf{n} = \frac{\bar{\mathbf{g}}_b}{|\bar{\mathbf{g}}_b|}. \tag{78}$$

Linearizing the anomaly field $(\nu_{\mathrm{OPH}} - 1)\mathbf{g}_b$ gives the susceptibility

$$A_{ij} = [\nu_{\mathrm{OPH}}(x) - 1]\delta_{ij} + x\nu'_{\mathrm{OPH}}(x)n_i n_j, \tag{79}$$

where

$$\nu'_{\mathrm{OPH}}(x) = -\frac{\lambda_{\mathrm{collar}} \exp[-\lambda_{\mathrm{collar}}\sqrt{x}]}{2\sqrt{x}[1 - \exp(-\lambda_{\mathrm{collar}}\sqrt{x})]^2}. \tag{80}$$

For a Fourier mode whose direction has cosine $\mu$ against $\mathbf{n}$, the density-response multiplier is

$$K_A^{(\rho)}(x, \mu) = \nu_{\mathrm{OPH}}(x) - 1 + x\nu'_{\mathrm{OPH}}(x)\mu^2. \tag{81}$$

An isotropic environmental average gives

$$\langle K_A^{(\rho)} \rangle = \nu_{\mathrm{OPH}}(x) - 1 + \frac{x}{3}\nu'_{\mathrm{OPH}}(x). \tag{82}$$

The contrast kernel used in perturbation equations is normalized by the background density ratio:

$$B_A(k, a) = \frac{\bar{\rho}_b(a)}{\bar{\rho}_A(a)} K_A^{(\rho)}(k, a). \tag{83}$$

In a realistic cosmological calculation, this local expression has to be integrated against the distribution of environmental fields,

$$K_A^{(\rho)}(k, a) = \int \mathrm{d}x\, \mathrm{d}\mu\, \Pi(x, \mu|k, a)[\nu_{\mathrm{OPH}}(x) - 1 + x\nu'_{\mathrm{OPH}}(x)\mu^2]. \tag{84}$$

This average is finite only under an explicit small-field support condition, for example

$$\int_0^{\epsilon} \mathrm{d}x\, \mathrm{d}\mu\, \Pi(x, \mu|k, a)\, x^{-1/2} < \infty \qquad \text{for some } \epsilon > 0. \tag{85}$$

If $\Pi$ has an exact atom at $x = 0$, the local nonzero-field kernel is not the correct FLRW response. The distribution $\Pi$ is an OPH finite-collar output or an explicitly declared environmental closure. Exact FLRW has $x = 0$, so (81) does not provide a regular universal CMB density-response kernel by itself.

The implementation contract uses one parent finite-collar functional:

$$\rho_{A,\mathrm{eq}}[X]c^2 = \frac{15}{8\pi^2 \ell(X)^4} \int_{\mathcal{C}_X} \mathrm{d}\mu_C\, I_{\omega_C}(A : D|B). \tag{86}$$

For finite samples $s$ with weights $w_s$, define

$$R(X) = \frac{\sum_s w_s I_s}{\sum_s w_s}, \tag{87}$$

$$K_A^{(\rho)}(k, a) = \frac{1}{\bar{\rho}_b} \frac{\partial \rho_{A,\mathrm{eq}}}{\partial \delta_b(k, a)}, \tag{88}$$

$$B_A(k, a) = \frac{\bar{\rho}_b(a)}{\bar{\rho}_A(a)} K_A^{(\rho)}(k, a). \tag{89}$$

The same evaluator emits the homogeneous equilibrium density, the linear density-response kernel, the settled galaxy source, and the cluster equilibrium source. A theorem-grade prediction requires the OPH collar measure $\mathrm{d}\mu_C$, the finite-screen ensemble, and the small-field support condition. Fitting $\Pi$ to CMB, weak-lensing, SPARC, or cluster data is excluded.

### 11.4 Compressed CMB, BAO, Growth, And $S_8$ Diagnostic

The local Boltzmann plumbing test passes the anomaly as a cold pressureless component with the flat-residual diagnostic parent-grid ratio $\rho_A/\rho_b = 5.363470441$. It uses CAMB 1.6.6, the OPH neutrino mass sum, and diagonal Gaussian compressed rows:

| Quantity | Prediction | Target | Pull |
|---|---|---|---|
| Planck $\Omega_m$ | 0.315905207 | $0.315 \pm 0.007$ | $+0.129$ |
| Planck $\sigma_8$ | 0.807787208 | $0.811 \pm 0.006$ | $-0.535$ |
| Planck $S_8$ | 0.828924043 | $0.831027707 \pm 0.0111$ | $-0.190$ |
| DESI DR1 BAO $\Omega_m$ | 0.315905207 | $0.295 \pm 0.015$ | $+1.394$ |
| DESI DR1 BAO/BBN/$\theta_*$ $H_0$ | 67.4 | $68.52 \pm 0.62$ | $-1.806$ |
| Weak-lensing $S_8$ | 0.828924043 | $0.790 \pm 0.016$ | $+2.433$ |

The diagonal compressed value is $\chi^2 = 11.463$ for six rows. This is a plumbing check. It does not replace Planck, DESI, or weak-lensing likelihoods. A publication-grade calculation needs the OPH $B_A(k, a)$ kernel in a custom Boltzmann module and the full experimental covariances.

## 12 Likelihood Contracts

### 12.1 SPARC Disk-Potential Contract

The publication-grade SPARC test must solve the conservative disk equation

$$\nabla^2\Phi = \nabla \cdot \left[\nu_\lambda\left(\frac{|\nabla\Phi_b|}{a_{0,\mathrm{OPH}}}\right)\nabla\Phi_b\right] \tag{90}$$

on an axisymmetric baryonic disk model. The hierarchy must marginalize or profile distance, inclination, disk and bulge mass-to-light ratios, gas scale, intrinsic scatter, velocity covariance, quality cuts, and grid convergence. The unit branch and the $\mathbb{Z}_6$/Poisson branch are fixed before this test; SPARC is used as a comparison, not as the selector for $\lambda_{\mathrm{collar}}$.

### 12.2 Cluster Map Contract

The cluster forward model uses gas hydrodynamics, stellar and galaxy tracers, transported defect stress, the repair source $\rho_{A,\mathrm{eq}}$, lensing projection, X-ray emissivity, SZ pressure, and covariance-aware map likelihoods. Merging clusters test whether the transported stress lags collisional gas. Relaxed clusters test whether the same stress law returns to the static equilibrium source. The static-only failure mode is part of the likelihood contract.

### 12.3 Boltzmann Contract

The Boltzmann module exposes

$$\bar\rho_A(a), \quad \bar\rho_{A,\mathrm{eq}}(a), \quad w_A(a), \quad c^2_{s,A}(k,a), \quad \sigma_A(k,a), \quad Q^\mu_A, \quad B_A(k,a), \quad \Gamma_{\mathrm{rec}}(k,a),$$

with the OPH neutrino mass sum in the same run. It must recover the ΛCDM cold-component limit when the exchange and stress corrections are turned off, reproduce the compressed diagnostic rows, and then run the full CMB TT/TE/EE, CMB lensing, BAO, supernova, weak-lensing, RSD, and $S_8$ likelihoods under the same nuisance model used for ΛCDM and $w_0w_a$.

# 13 Numerical Galaxy Comparison

The OPH capacity benchmark used here is

$$a_{0,\mathrm{OPH}} = 1.029186271 \times 10^{-10}\,\mathrm{m\,s^{-2}}. \tag{91}$$

The common empirical reference is

$$a_0^{\mathrm{emp}} = 1.2 \times 10^{-10}\,\mathrm{m\,s^{-2}}. \tag{92}$$

The $\mathbb{Z}_6$/Poisson branch gives

$$\lambda_{\mathrm{collar}} = 0.934300639, \qquad a_{0,\mathrm{eff}} = 1.179018696 \times 10^{-10}\,\mathrm{m\,s^{-2}}. \tag{93}$$

| Quantity | Value | Comparison |
|---|---|---|
| $a_{0,\mathrm{OPH}}$ | $1.029186271 \times 10^{-10}$ | −8.070% versus best same-function RAR |
| $a_{0,\mathrm{OPH}}$ | $1.029186271 \times 10^{-10}$ | −14.234% versus empirical reference |
| $a_{0,\mathrm{eff}}$, $\mathbb{Z}_6$/Poisson | $1.179018696 \times 10^{-10}$ | +5.314% versus best same-function RAR |
| $a_{0,\mathrm{eff}}$, $\mathbb{Z}_6$/Poisson | $1.179018696 \times 10^{-10}$ | −1.748% versus empirical reference |

## 13.1 SPARC RAR

Using the public SPARC RAR data [7, 8], the local diagnostic gives:

| Scenario | $a_0$ in $\mathrm{m\,s^{-2}}$ | RMS dex | Binned RMS dex |
|---|---|---|---|
| OPH unit | $1.029186271 \times 10^{-10}$ | 0.134208 | 0.026169 |
| $\mathbb{Z}_6$/Poisson | $1.179018696 \times 10^{-10}$ | 0.132834 | 0.015760 |
| Empirical reference | $1.200000000 \times 10^{-10}$ | 0.132913 | 0.016164 |
| Best same-function | $1.119530397 \times 10^{-10}$ | 0.132947 | 0.017428 |

## 13.2 Rotation Diagnostics

The fixed stellar mass-to-light diagnostic and the first nuisance-profiled diagnostic give:

| Scenario | Fixed-M/L RMS | Profiled-M/L $\chi^2$/pt | Profiled RMS |
|---|---|---|---|
| OPH unit | 23.366208 | 1.341948 | 12.420181 |
| $\mathbb{Z}_6$/Poisson | 22.868866 | 1.390941 | 12.590203 |
| Empirical reference | 22.842145 | 1.400930 | 12.624079 |

Units for the RMS columns are $\mathrm{km\,s^{-1}}$.

## 13.3 Systematic Likelihood Scaffold

A stronger local scaffold profiles stellar M/L, distance, inclination, and gas scale per galaxy using the SPARC sample table [6, 7]. It uses $Q \leq 2$, inclination at least 30°, and 3168 rotation-curve rows.

| Scenario | $\chi^2$/pt | $\chi^2$/dof | RMS | Median disk M/L |
|---|---|---|---|---|
| OPH unit | 1.016589 | 1.275672 | 11.249541 | 0.519102 |
| $\mathbb{Z}_6$/Poisson | 1.021093 | 1.281324 | 11.276832 | 0.497435 |
| Empirical reference | 1.023261 | 1.284045 | 11.285518 | 0.493865 |

The systematic scaffold mildly favors the unit branch. The $\mathbb{Z}_6$/Poisson branch remains the better acceleration-scale and fixed-M/L RAR match. The fixed-M/L improvement alone is weak evidence for the $\mathbb{Z}_6$ bridge theorem.

### 13.4 Empirical Implementation Scorecard

The executable empirical pass combines four diagnostic layers: public SPARC tables, the flat capacity-saturated homogeneous anomaly state, the finite repair matrix with a cluster timing gate, and the compressed CAMB rows. The single-pass output is:

| Quantity | OPH output | Measurement comparison |
|---|---|---|
| $\Omega_A$ | 0.264114401 | Planck-like matter budget |
| $\Omega_{\Lambda,\mathrm{OPH}}$ | 0.684423332 | capacity de Sitter branch |
| $\rho_A/\rho_b$ | 5.363470441 | flat residual ratio |
| $\gamma_{\mathrm{rec}}$ | 0.050717471 | finite repair matrix gap |
| $a_{0,\mathrm{OPH}}$ | $1.029186271 \times 10^{-10}$ | 8.070% low versus best RAR |
| $a_{0,\mathrm{eff}}$, $\mathbb{Z}_6$/Poisson | $1.179018696 \times 10^{-10}$ | 1.748% low versus common empirical $a_0$ |
| SPARC systematic $\chi^2$/pt, unit | 1.016589 | best scaffold row |
| SPARC systematic $\chi^2$/pt, $\mathbb{Z}_6$ | 1.021093 | close scaffold row |
| CAMB $\Omega_m$ | 0.315905206 | $+0.129\sigma$ versus Planck |
| CAMB $\sigma_8$ | 0.807787204 | $-0.535\sigma$ versus Planck |
| CAMB $S_8$ | 0.828924037 | $+2.433\sigma$ versus weak lensing |

The compressed CMB/BAO/growth score is

$$\chi^2_{\mathrm{diag}} = 11.463 \qquad \text{for six compressed rows.}$$

The pressure point is weak-lensing $S_8$. The diagnostic value is 0.828924037. The DES/KiDS compressed row is $0.790 \pm 0.016$.

For the cluster timing gate, take a 200 kpc initial anomaly separation, a 0.2 Gyr time after passage, and an example measured offset $150 \pm 50$ kpc. The repair matrix gives:

| $\tau_{\mathrm{rec}}$ in Gyr | $t_{\mathrm{commit}}$ in Myr | Retained fraction | Offset in kpc | $\chi^2$ |
|---|---|---|---|---|
| 0.3 | 15.2152 | 0.513417 | 102.683 | 0.896 |
| 0.5 | 25.3587 | 0.670320 | 134.064 | 0.102 |
| 1.0 | 50.7175 | 0.818731 | 163.746 | 0.076 |

This row is a timing gate. Map likelihoods need observed lensing maps, gas maps, stellar maps, merger-age priors, and covariance.

### 13.5 Implementation Work Packages

The code required for paper-grade empirical tests consists of the following modules:

1. An axisymmetric conservative disk-potential solver for

$$\nabla^2 \Phi = \nabla \cdot [\nu(|\nabla \Phi_b|/a_{0,\mathrm{OPH}}) \nabla \Phi_b],$$

with grid-convergence tests and lensing-potential output.

2. A hierarchical SPARC likelihood that samples or profiles distance, inclination, disk M/L, bulge M/L, gas scale, intrinsic scatter, velocity covariance, and selection cuts under one nuisance model.

3. A finite-collar parent evaluator for $\rho_A(a)$, $K_A^{(\rho)}(k,a)$, $B_A(k,a)$, and $\rho_{A,\mathrm{eq}}[X]$, using OPH collar samples as input. The scalar-load diagnostic grid is plumbing only.

4. A CAMB or CLASS anomaly module exposing the background density, stress variables, exchange current, relaxation kernel, environmental response kernel, and the OPH neutrino mass sum in one run.

5. A cluster forward model with gas hydrodynamics, galaxy and stellar tracers, transported anomaly stress, lensing projection, X-ray and SZ observables, merger-age priors, and map covariance.

6. A likelihood runner for Planck or ACT spectra, CMB lensing, BAO, supernovae, weak lensing, RSD, SPARC, and cluster maps, with model comparison against ΛCDM, empirical MOND functions, halo models, and $w_0 w_a$ extensions.

7. A reproducibility harness with fixed data hashes, fixed random seeds, unit checks, and independent numerical replication of the scorecard rows.

### 13.6 Correction Audit

The comparison mostly tests normalization because the OPH static law uses the same acceleration-family shape as the empirical RAR fit. If $a_{0,\mathrm{OPH}}$ is held fixed, different targets imply different $\lambda_{\mathrm{collar}}$ values:

| Target | Required $\lambda_{\mathrm{collar}}$ | Reserve $-\ln\lambda_{\mathrm{collar}}$ | Ratio to $P/24$ |
|---|---|---|---|
| Best same-function RAR | 0.958802256 | 0.042070424 | 0.619074 |
| Binned RAR | 0.936737944 | 0.065351711 | 0.961663 |
| Common empirical $a_0$ | 0.926096769 | 0.076776547 | 1.129781 |
| Fixed-M/L rotation | 0.926654674 | 0.076174302 | 1.120919 |
| OPH $\mathbb{Z}_6$/Poisson | 0.934300639 | 0.067957009 | 1.000000 |

This rules out several tempting shortcuts. In the normalization-profiled all-point objective, a generalized activation exponent near 0.503420 appears. With fixed $a_{0,\mathrm{OPH}}$, direct RAR objectives give exponents near 0.516 all-point and 0.519 weighted-bin. These shifts are objective-dependent, and flat deep-IR rotation requires 0.5. Moving the exponent would buy little and would damage the BTFR theorem.

Finite-thickness nonuniformity can raise $\lambda_{\mathrm{collar}}$ above $\exp(-P/24)$, which moves the coefficient toward the binned RAR value. It cannot lower $\lambda_{\mathrm{collar}}$ toward the common empirical or fixed-M/L values while the mean reserve remains $P/24$ and Poisson counting is retained. An exact hit to the common empirical acceleration would require about 13% more protected inactive reserve than $P/24$, or a different microscopic selector. That extra reserve needs an OPH theorem before use.

## 14 Claim Boundaries

| Pressure point | OPH route | Status |
|---|---|---|
| Unknown particle | Dark source is a carried modular/collar information-defect stress. | Conditional source target. |
| Galaxy RAR and BTFR | Collar/cut activation gives the square-root transition and Poisson activation law under independent-increment repair counting. | Conditional static theorem. |
| Acceleration scale | Capacity scale gives $a_{0,\mathrm{OPH}}$; $\mathbb{Z}_6$/Poisson reserve gives $a_{0,\mathrm{eff}}$ close to the empirical scale. | Conditional coefficient theorem. |
| Galaxy lensing | No-slip branch gives $\Phi = \Psi$ if the information-defect stress is dust on settled galaxies. | Conditional stress theorem. |
| Cluster offsets | Transported anomaly can lag collisional gas; the Jacobi clock gives a no-fit scale under the tidal-mismatch premise. | Forward likelihood contract. |
| CMB and growth | Flat capacity state selection gives a CDM-like homogeneous residual; the closed branch has a conserved load current; environmental kernels describe nonzero-field response. | Amplitude selector and real Boltzmann likelihood work in progress. |

The supported claim is:

> OPH defines an imperfect-information correction to the recovered Einstein branch. Under explicit bridge premises, the static galaxy equation reproduces the RAR/BTFR functional form from collar/cut repair statistics and predicts an acceleration scale close to SPARC. The unit branch is low in normalization. The $\mathbb{Z}_6$/Poisson branch lands within 1.748% of the common empirical acceleration reference. Profiled stellar, distance, inclination, and gas nuisances mildly favor the unit branch. The Jacobi clock gives a no-fit cluster-offset scale under its tidal-mismatch premise. Flat capacity state selection gives a Planck-like homogeneous information-defect residual, while closed-branch transport gives a conserved load current. Cluster and CMB/growth likelihoods require measured cluster covariances, a finite-screen amplitude selector, the environmental kernel, and a Boltzmann implementation.

Claim outside this paper:

> OPH has fully derived dark matter and solved MOND's cluster and CMB problems.

That stronger statement requires SPARC publication likelihoods, cluster likelihoods, and Boltzmann likelihoods.

## 15 Proof Ledger

| Target | OPH closure | Numerical state |
|---|---|---|
| Galaxy source | Minimal scalar recoverability defect $R_C = I(A : D\|B)$. | Conditional source target. |
| Quotient-edge locality | Scalar repair opportunities are edge-center cut-register events on the physical quotient. | Conditional bridge. |
| Finite thickness | Product transverse regulator with one quotient trace makes the protected center reserve uniform through the weighted collar. | Conditional bridge. |
| No-slip stress | Positive kinetic collar measure; settled monokinetic branch has dust stress and vanishing information-sector anisotropic stress. | Conditional stress theorem. |
| Repair relaxation | Fixed repair-commit chain gives $\tau_{\rm rec} = t_{\rm commit}/\gamma_{\rm rec}$. | Conditional packet theorem and clock candidate. |
| CMB and growth | Nonzero-environment kernel is explicit; exact FLRW static RAR kernel is blocked by a no-go theorem. | Boltzmann module contract. |
| Abundance history | Flat capacity state selection gives a CDM-like residual; the closed transported branch has a conserved load current and charge. | Amplitude selector work in progress. |
| Linear transfer | The perturbation equation has a relaxation transfer solution between CDM-like and tracking limits. | Requires evaluated $B_A(k, a)$. |
| Parent functional | The finite collar expectation of the positive repair defect emits $\rho_{A,\rm eq}$ and $B_A$ if its FLRW limit is regular. | Finite-sample evaluator contract. |
| Repair matrix | A reversible Metropolis kernel on packet states gives a concrete $K$. | Conditional on declared packet schema. |
| Jacobi clock | The tidal scalar $(E_{ij}E^{ij}/6)^{1/4}$ gives a no-fit dynamic repair scale under the Jacobi mismatch premise. | Conditional clock. |

### 15.1 Positive Galaxy Source

**Theorem target 0A.** Let a settled galaxy collar state be judged by a scalar repair defect $D_C$ with these properties: $D_C \geq 0$; $D_C = 0$ exactly on locally recoverable Markov states; $D_C$ is invariant under boundary redundancy; $D_C$ is additive over independent collar components; and $D_C$ is monotone under local recovery maps. Then the minimal scalar branch is

$$D_C = \kappa I(A : D|B), \qquad \kappa \geq 0. \tag{94}$$

The OPH dark branch uses $R_C = I(A : D|B)$.

**Explicit premise.** The galaxy-scale finite-stage modular nonadditivity is represented by this minimal scalar recoverability defect on settled collar states.

**Conditional argument.** Conditional mutual information has all listed properties on a finite collar. A different scalar with the same zero set and additivity would add an independent scalar datum to the minimal branch. The minimal scalar selector is therefore fixed up to a nonnegative multiplier. The OPH unit convention sets that multiplier to one. Since $I(A : D|B) \geq 0$, the weak-field density defined by the anomaly normalization is nonnegative. □

**Proof obligation.** Derive the map from the finite-stage modular nonadditivity term to $I(A : D|B)$ on the galaxy branch.

**Failure consequence.** If the modular remainder is not this positive defect, the attractive source and normalization formula must be replaced by the derived scalar.

## 15.2 Quotient-Edge Scalar Opportunity

**Theorem target 0B.** At fixed cutoff, physical collar observables live on the quotient-local fixed-point algebra. A settled galaxy scalar repair opportunity is the local event that the quotient-local recoverability defect on a collar cut is nonzero. Then the scalar opportunity is counted on the edge-center cut register.

**Explicit premise.** The scalar recoverability event has no independent bulk, screen-area, or non-central collar channel.

**Conditional argument.** The defect is a physical scalar. Boundary-redundant representatives produce the same defect value because the quotient repair law descends through the projection onto physical collar states. The event that the defect is nonzero is therefore an observable event on the quotient-local collar algebra. For a connected cut, the fixed-cutoff edge-center decomposition supplies the central projectors that resolve physical cut sectors. A scalar event with no vector, tensor, or bulk label selects support through those central projectors. Thus the scalar opportunity count is a quotient-edge cut-register count. The protected $\mathbb{Z}_6$ center reserve is supported on the same register. □

**Proof obligation.** Prove the fixed-cutoff uniqueness statement: every quotient-local scalar recoverability event is resolved on the edge-center cut register.

**Failure consequence.** If a separate scalar channel exists, the $\mathbb{Z}_6$ reserve does not fix $\lambda_{\text{collar}}$ without an additional co-registration theorem.

## 15.3 Uniform Finite-Thickness Reserve

**Theorem target 0C.** Let the finite collar be a transverse thickening of one quotient-edge cut algebra with normalized transverse weight $w(y)$ and a single quotient trace on the edge algebra. Let the protected $\mathbb{Z}_6$ reserve be a central edge projector whose mean on that trace is $\epsilon_{\mathbb{Z}_6} = P/24$. Then

$$\lambda_{\text{collar}} = \exp(-P/24). \tag{95}$$

**Explicit premise.** The scalar-weighted transverse regulator is a product thickening with one quotient trace and uniform protected-center density.

**Conditional argument.** The protected reserve belongs to the edge-center algebra. The transverse coordinate belongs to the regulator thickening. With one quotient trace on the edge algebra, the trace of the protected central projector is the same on every transverse slice that carries scalar activation weight. Hence $\epsilon_{\mathbb{Z}_6}(y) = \epsilon_{\mathbb{Z}_6}$. Poisson zero-reserve survival gives $\exp(-\epsilon_{\mathbb{Z}_6})$ at each slice. Averaging with $w(y)$ leaves the same value. Using $\epsilon_{\mathbb{Z}_6} = P/24$ gives the stated coefficient. □

**Proof obligation.** Derive $w(y)$, the transverse regulator, and the quotient trace from base OPH screen/collar data.

**Failure consequence.** If the reserve density is nonuniform, the coefficient is

$$\int \mathrm{d}y\, w(y) \exp[-\epsilon_{\mathbb{Z}_6}(y)],$$

not necessarily $\exp(-P/24)$.

### 15.4 No-Slip Settled Stress

**Theorem target 0D.** Let the information-defect stress be the second moment of a positive finite-collar kinetic measure $\mathrm{d}\mathcal{E}_x(v)$ on the future unit timelike shell:

$$T^{ab}_{\mathcal{I}} = \int v^a v^b \,\mathrm{d}\mathcal{E}_x(v). \tag{96}$$

If the settled galaxy branch is monokinetic in its rest frame, then

$$T^{ab}_{\mathcal{I}} = \rho_{\mathcal{I}} u^a_{\mathcal{I}} u^b_{\mathcal{I}}, \qquad P_{\mathcal{I}} = 0, \qquad q^a_{\mathcal{I}} = 0, \qquad \pi^{ab}_{\mathcal{I}} = 0. \tag{97}$$

For negligible or separately modeled anisotropic stress in all other sectors, $\Phi = \Psi$.

**Explicit premise.** The finite collar packet state emits the positive kinetic measure, and the settled galaxy branch is monokinetic at the coarse scale used for lensing.

**Conditional argument.** A positive measure on future timelike velocities has a symmetric future-positive second moment. In the monokinetic branch the measure is concentrated at one four-velocity, so the second moment is $\rho_{\mathcal{I}} u^a_{\mathcal{I}} u^b_{\mathcal{I}}$. Its pressure, heat flux, and anisotropic stress vanish in that rest frame. The off-diagonal spatial Einstein equation then gives $\Phi = \Psi$ when the other sectors carry no uncompensated anisotropic stress. □

**Proof obligation.** Derive $\mathrm{d}\mathcal{E}_x(v)$ from the finite OPH collar packet state or from an equivalent covariant parent action.

**Failure consequence.** If pressure or anisotropic stress survives coarse graining, the lensing slip equation must include those terms and no-slip is regime-dependent.

### 15.5 Repair Relaxation Scale

**Theorem 0E.** Let $K$ be the fixed-cutoff repair-commit transition matrix for one local coarse cell at fixed baryonic source. If $K$ is finite, irreducible, and aperiodic, with spectral gap $\gamma_{\rm rec}$, then

$$\tau_{\rm rec} = \frac{t_{\rm commit}}{\gamma_{\rm rec}}. \tag{98}$$

If every allowed repair move has probability at least $q_{\rm min} > 0$ and the comparison chain has conductance $\Phi_K$, then

$$\gamma_{\rm rec} \geq \frac{1}{2} q_{\rm min} \Phi_K^2. \tag{99}$$

**Proof.** A finite irreducible aperiodic chain has a unique stationary state. Every nonstationary mode decays as $\exp(-\gamma_n n)$ in repair-step time, and the slowest nonzero mode has rate $\gamma_{\rm rec}$. One physical commit step takes $t_{\rm commit}$, so physical relaxation has rate $\gamma_{\rm rec}/t_{\rm commit}$. The conductance bound follows from the standard Cheeger comparison for the reversible comparison chain and the lower move probability $q_{\rm min}$. □

### 15.6 Environmental Kernel Around A Nonzero Field

**Conditional theorem 0F.** Let the settled anomaly field be

$$\mathbf{g}_A = (\nu_{\rm OPH} - 1)\mathbf{g}_b. \tag{100}$$

Linearize around a nonzero background field $\bar{\mathbf{g}}_b$. With

$$x = |\bar{\mathbf{g}}_b|/a_{0,\rm OPH}, \qquad n_i = \bar{g}_{b,i}/|\bar{\mathbf{g}}_b|,$$

the Fourier density response to a baryonic density mode is

$$K_A^{(\rho)}(x,\mu) = \nu_{\rm OPH}(x) - 1 + x\nu'_{\rm OPH}(x)\mu^2, \tag{101}$$

where $\mu$ is the cosine between the mode direction and $\mathbf{n}$. The contrast kernel is

$$B_A(k,a) = \frac{\bar{\rho}_b(a)}{\bar{\rho}_A(a)} K_A^{(\rho)}(k,a). \tag{102}$$

**Proof.** The first variation is

$$\delta g_{A,i} = \left([\nu_{\rm OPH}(x) - 1]\delta_{ij} + x\nu'_{\rm OPH}(x) n_i n_j\right) \delta g_{b,j}. \tag{103}$$

Taking the divergence and using the Fourier-space Poisson relation for $\delta\mathbf{g}_b$ gives the displayed scalar multiplier. The derivative of

$$\nu_{\rm OPH}(x) = [1 - \exp(-\lambda_{\rm collar}\sqrt{x})]^{-1}$$

is

$$\nu'_{\rm OPH}(x) = -\frac{\lambda_{\rm collar}\exp[-\lambda_{\rm collar}\sqrt{x}]}{2\sqrt{x}[1-\exp(-\lambda_{\rm collar}\sqrt{x})]^2}.$$

□

### 15.7 Background Abundance Green Function

**Theorem 0G.** Let the background anomaly density obey

$$\rho_A' + 3\mathcal{H}\rho_A = -a\Gamma_{\rm rec}(\rho_A - \rho_{A,\rm eq}). \tag{104}$$

Define the comoving densities

$$R_A = a^3\rho_A, \qquad R_{A,\rm eq} = a^3\rho_{A,\rm eq}. \tag{105}$$

Then

$$R_A(\eta) = W(\eta,\eta_i)R_A(\eta_i) + \int_{\eta_i}^{\eta} \mathrm{d}\eta'\, W(\eta,\eta')a(\eta')\Gamma_{\rm rec}(\eta')R_{A,\rm eq}(\eta'), \tag{106}$$

where

$$W(\eta,\eta') = \exp\left[-\int_{\eta'}^{\eta} \mathrm{d}\tilde\eta\, a(\tilde\eta)\Gamma_{\rm rec}(\tilde\eta)\right]. \tag{107}$$

**Proof.** Multiplying the background equation by $a^3$ gives

$$R_A' = -a\Gamma_{\rm rec}(R_A - R_{A,\rm eq}). \tag{108}$$

This is a first-order linear equation. Multiplication by the integrating factor

$$\exp\left[\int_{\eta_i}^{\eta} \mathrm{d}\eta'\, a(\eta')\Gamma_{\rm rec}(\eta')\right] \tag{109}$$

and integration gives the stated expression. □

The theorem shows exactly where abundance information enters: the initial comoving anomaly load, the equilibrium source, and the repair rate. These are OPH microphysical outputs, not SPARC parameters.

### 15.8 Linear Relaxation Transfer

**Theorem 0H.** For fixed $k$, assume the no-slip anomaly perturbation equation

$$\delta_A' = -\theta_A + 3\Phi' - a\Gamma_{\rm rec}q_A(\delta_A - B_A\delta_b). \tag{110}$$

Define

$$\Xi_A = a\Gamma_{\rm rec}q_A. \tag{111}$$

Then

$$\begin{aligned}\delta_A(\eta,k) &= W_A(\eta,\eta_i)\delta_A(\eta_i,k)\\ &\quad + \int_{\eta_i}^{\eta} \mathrm{d}\eta'\, W_A(\eta,\eta')\left[-\theta_A + 3\Phi' + \Xi_A B_A\delta_b\right]_{\eta',k},\end{aligned} \tag{112}$$

with

$$W_A(\eta,\eta') = \exp\left[-\int_{\eta'}^{\eta} \mathrm{d}\tilde\eta\, \Xi_A(\tilde\eta,k)\right]. \tag{113}$$

**Proof.** Move the damping term to the left:

$$\delta_A' + \Xi_A \delta_A = -\theta_A + 3\Phi' + \Xi_A B_A \delta_b. \tag{114}$$

The integrating-factor solution is the displayed expression. □

This formula makes the two useful limits explicit. If $\Xi_A/\mathcal{H} \ll 1$, the anomaly evolves like pressureless free-falling matter. If $\Xi_A/\mathcal{H} \gg 1$, it tracks the OPH equilibrium source up to derivative corrections. Cluster offsets and settled galaxies require the same relaxation operator to sit between those limits.

### 15.9 Static-To-Linear Separation

**Theorem 0I.** The nonlinear static RAR law

$$g_{\text{obs}} = \frac{g_b}{1 - \exp[-\lambda_{\text{collar}} \sqrt{g_b/a_{0,\text{OPH}}}]} \tag{115}$$

does not define the FLRW anomaly abundance or the linear CMB kernel by itself.

**Proof.** The RAR law is written for a static weak-field acceleration vector sourced by a resolved baryonic inhomogeneity. An exactly homogeneous FLRW background has no preferred spatial acceleration vector. Its scalar data are background densities and perturbation kernels, with no radial acceleration profile. In addition, the RAR map contains $\sqrt{g_b}$, so its first derivative is singular at $g_b = 0$ if treated as a homogeneous-background Taylor series. Therefore the static law cannot be linearized into CMB equations without an OPH parent functional. The parent functional must emit $\rho_A(a)$ and $B_A(k, a)$. □

### 15.10 Parent Collar Functional

**Theorem target 0J.** Let $\mathcal{C}_x(a)$ be the finite collar family contributing to a coarse FLRW cell $x$ at scale factor $a$. Assume this finite-collar state selection has a regular FLRW limit. Let $\omega[\rho_b]$ be the OPH collar state selected by the baryonic source, and let

$$R_C[\omega] = I_\omega(A : D|B). \tag{116}$$

Define

$$\rho_{A,\text{eq}}(x, a)c^2 = \frac{15}{8\pi^2 \ell(a)^4} \int_{\mathcal{C}_x(a)} \mathrm{d}\mu_C \, R_C[\omega[\rho_b]]. \tag{117}$$

Then (117) is a scalar parent collar functional that emits a background equilibrium source and a linear kernel:

$$\bar{\rho}_{A,\text{eq}}(a) = \rho_{A,\text{eq}}[\bar{\rho}_b](a), \tag{118}$$

$$K_A^{(\rho)}(k, a) = \widehat{\frac{\delta \rho_{A,\text{eq}}(x, a)}{\delta \rho_b(x', a)}}\Bigg|_{\bar{\rho}_b}(k), \qquad B_A(k, a) = \frac{\rho_b(a)}{\rho_A(a)} K_A^{(\rho)}(k, a). \tag{119}$$

**Conditional argument.** The collar measure, the conditional mutual information, and the selected collar state are quotient-local scalar data. The integral is therefore a scalar density on the coarse FLRW cell. Evaluating the functional on a homogeneous baryonic density gives the background equilibrium source. Taking the first functional derivative around that homogeneous state gives a translation-invariant and rotation-invariant linear response kernel. Its Fourier transform is $K_A^{(\rho)}(k, a)$, and division by the background density gives $B_A(k, a)$. □

The finite-collar evaluation of (117) is the mathematical target for replacing a fitted dark-matter abundance.

**Proof obligation.** Evaluate the collar measure and selected state from finite OPH screen/collar data in the homogeneous, perturbative, galaxy, and cluster regimes.

### 15.11 Finite Repair Matrix

**Theorem 0K.** Let $\mathsf{S}$ be the finite packet-state space declared by one fixed-cutoff overlap interface. Let $\pi_{\text{eq}}(s|\rho_b)$ be the equilibrium packet distribution emitted by the parent collar functional. Let $q(s, s')$ be an inverse-closed local proposal rule generated by the declared repair menu, with $q(s, s') > 0$ iff $q(s', s) > 0$. Define, for $s \neq s'$,

$$K(s, s') = q(s, s') \min\left(1, \frac{\pi_{\text{eq}}(s'|\rho_b) q(s', s)}{\pi_{\text{eq}}(s|\rho_b) q(s, s')}\right), \tag{120}$$

and

$$K(s, s) = 1 - \sum_{s' \neq s} K(s, s'). \tag{121}$$

Then $K$ is a finite stochastic repair transition matrix with stationary distribution $\pi_{\text{eq}}$. If the proposal graph is connected and some rejection or hold probability is nonzero, $K$ is irreducible and aperiodic.

**Proof.** Each row sums to one by construction, so $K$ is stochastic. For $s \neq s'$, inverse-closed proposals make both directions available. Hence

$$\pi_{\text{eq}}(s) K(s, s') = \min\big(\pi_{\text{eq}}(s) q(s, s'), \pi_{\text{eq}}(s') q(s', s)\big) = \pi_{\text{eq}}(s') K(s', s). \tag{122}$$

Detailed balance proves stationarity. Connected proposal support gives irreducibility, and a nonzero hold probability gives aperiodicity. □

This construction declares $K$ from OPH data: packet states, local proposals, and the parent equilibrium distribution.

**Scalar-load diagnostic.** The executable packet diagnostic takes

$$\mathsf{S} = \{0, 1, \ldots, 40\}, \qquad \pi_{\text{eq}}(n) = \text{Pois}(n; 5.363470441)$$

with truncation at 40, nearest-neighbor inverse-closed proposals, and hold probability 0.25. The resulting matrix has row-sum error $1.110 \times 10^{-16}$, detailed-balance error $6.939 \times 10^{-18}$, and

$$\gamma_{\text{rec}} = 0.050717471.$$

This diagnostic is a finite packet schema. The OPH task is to derive that schema, or its replacement, from the finite collar states.

### 15.12 Finite Repair-Graph Gap

**Theorem 0L.** For a declared fixed-cutoff repair graph with transition matrix $K$, the relaxation time used by clusters and perturbations is computable from the second eigenvalue. If $K$ is reversible with stationary distribution $\pi_{\text{eq}}$, then

$$\gamma_{\text{rec}} = 1 - \lambda_2(K) \tag{123}$$

in discrete repair-step units, where $\lambda_2$ is the largest nontrivial eigenvalue in absolute value on the mean-zero subspace. For continuous-time generator $L = K - \mathbf{1}$, the gap is the smallest positive eigenvalue of $-L$.

**Proof.** Reversibility makes $K$ self-adjoint in $L^2(\pi_{\rm eq})$. The stationary vector has eigenvalue one. Every mean-zero mode decomposes into eigenvectors and decays by its eigenvalue per repair step. The slowest decaying nonstationary mode is therefore the largest nontrivial eigenvalue, giving $\gamma_{\rm rec} = 1 - \lambda_2(K)$. The continuous-time statement follows by exponentiating the generator. □

This is an exact finite-matrix target for the repair/commit microphysics. A cluster timescale is a prediction only after this matrix or its OPH state functional is supplied.

### 15.13 Jacobi Repair Clock

**Theorem target 0M.** Assume repair timing is set by the overlap-visible Jacobi mismatch rate of transported information-defect records. Let $E_{ij} = C_{0i0j}$ be the electric Weyl tensor in the local free-fall frame, equivalently the trace-free Newtonian tidal tensor in the weak-field limit. Homogeneous FLRW Ricci focusing is excluded from this local cluster-offset clock. The scalar repair rate is

$$\Gamma_J = \left( \frac{E_{ij}E^{ij}}{6} \right)^{1/4}. \tag{124}$$

For a Newtonian point source this gives

$$\Gamma_J = \sqrt{\frac{GM}{r^3}}, \qquad \tau_J = \sqrt{\frac{r^3}{GM}}. \tag{125}$$

**Explicit premise.** The repair-cycle clock is controlled by overlap-visible geodesic-deviation mismatch of transported information-defect records.

**Conditional argument.** The relative acceleration of neighboring transported records is controlled by the Jacobi operator. The local cluster-offset part is the trace-free Weyl electric tensor $E_{ij}$; the homogeneous FLRW Ricci term changes the background expansion and is excluded from the local offset between collisional gas and transported record stress. A scalar clock made from $E_{ij}$, with no chosen direction and no extra dimensional constant, must be a power of $E_{ij}E^{ij}$. The fourth root has units of inverse time. For a point Newtonian source the eigenvalues of $E_{ij}$ are $-2GM/r^3, GM/r^3, GM/r^3$ up to sign convention, hence $E_{ij}E^{ij} = 6(GM/r^3)^2$. The normalization in the theorem makes the point-source result exact. □

**Proof obligation.** Derive from OPH packet microphysics that the commit clock is the overlap-visible Weyl-Jacobi mismatch clock, while pixel, de Sitter, entropy-production, local-acceleration, and Ricci-curvature clocks are excluded. The self-consistent prediction evaluates $E_{ij}$ from the total weak-field potential determined by baryons plus information-defect stress.

**Failure consequence.** Without that derivation, $\Gamma_J$ is a no-fit dynamic diagnostic rather than a repair-clock theorem.

**Static-clock boundary.** The static equilibrium equation contains $\rho_{A,\rm eq}$ but no relaxation rate. Replacing $\Gamma_{\rm rec}$ by any positive function leaves the same settled solution. The Jacobi clock is therefore a dynamic OPH premise and has no theorem status inside the static RAR law.

### 15.14 Homogeneous Anomaly State Selection

**Theorem target 0N.** Under flat capacity-saturated FLRW state selection, the homogeneous anomaly abundance is

$$\Omega_{A0} = 1 - \Omega_{\Lambda,\mathrm{OPH}} - \Omega_{b0} - \Omega_{\nu 0} - \Omega_{r0}, \qquad \Omega_{\Lambda,\mathrm{OPH}} = \left(\frac{H_{\mathrm{dS}}}{H_0}\right)^2. \tag{126}$$

If the homogeneous anomaly charge is transported as dust with no homogeneous repair exchange, then

$$\rho_A(a) = \rho_{A0} a^{-3}. \tag{127}$$

**Explicit premise.** The homogeneous state is flat, capacity-saturated, and selected by a residual Friedmann constraint after $H_0$, baryons, neutrinos, and radiation are supplied.

**Conditional argument.** The flat Friedmann equation at $a = 1$ is

$$1 = \Omega_{\Lambda,\mathrm{OPH}} + \Omega_{b0} + \Omega_{\nu 0} + \Omega_{r0} + \Omega_{A0}. \tag{128}$$

Solving for $\Omega_{A0}$ gives the displayed residual. Pressureless homogeneous transport with zero homogeneous exchange obeys

$$\rho_A' + 3\mathcal{H}\rho_A = 0,$$

whose solution is $\rho_A(a) = \rho_{A0} a^{-3}$. $\square$

**Proof obligation.** Derive the conserved homogeneous information-defect charge $Q_A$ from OPH screen state data, or declare it as cosmological state data.

**Failure consequence.** The CMB dark abundance is conditional if $Q_A$ is not selected by OPH microphysics.

**Capacity boundary.** The screen capacity fixes the de Sitter scale. After $H_0$ is supplied, it fixes $\Omega_{\Lambda,\mathrm{OPH}}$. It does not by itself fix the baryon density, the neutrino density, the radiation density, or a conserved homogeneous anomaly charge. A standalone OPH cosmology needs a theorem of the form

$$Q_A = F_{\mathrm{OPH}}(\text{screen capacity}, P, \text{screen state data})$$

or an explicitly declared state-selection premise such as (72).

## 16 Falsifiers

The OPH dark branch fails if any of the following happens:

1. OPH collar/cut microphysics cannot derive the activation law $p(x) = 1 - \exp[-\lambda_{\mathrm{collar}}\sqrt{x}]$, or a replacement law that passes the same galaxy tests.

2. The carried galaxy defect is not positive or not attractive.

3. The settled anomaly stress cannot give $\Phi = \Psi$.

4. A full SPARC likelihood rejects both the unit branch and every independently derived coefficient branch under the same nuisance treatment used for comparison models.
5. Cluster lensing offsets require a relaxation law incompatible with old settled-galaxy RAR.
6. CMB peaks, CMB lensing, BAO, weak lensing, or growth reject every $\Gamma_{\rm rec}$, $B_A(k, a)$, and $\rho_A(a)$ closure compatible with the galaxy branch.

## 17 Reproducibility

The numerical tables were produced in the OPH cosmology workspace by:

```
python3 cosmology/scripts/dark_sector_measurement_ledger.py
python3 cosmology/scripts/dark_empirical_scorecard.py --quiet
python3 cosmology/scripts/sparc_systematic_likelihood.py
python3 cosmology/scripts/dark_perturbation_parameters.py
python3 cosmology/scripts/dark_repair_transition_matrix.py
python3 cosmology/scripts/dark_cluster_boltzmann_prelikelihood.py
python3 cosmology/scripts/dark_jacobi_repair_clock.py
python3 cosmology/scripts/dark_homogeneous_state_selection.py
python3 cosmology/scripts/dark_parent_collar_grid.py --diagnostic-scalar-load \
  --mu-eq 5.363470440729118 \
  --out cosmology/outputs/dark_parent_flat_capacity_state_selection.json
python3 cosmology/scripts/dark_cmb_bao_growth_s8_likelihood.py \
  --parent-grid cosmology/outputs/dark_parent_flat_capacity_state_selection.json
python3 cosmology/scripts/camb_fixed_neutrino_compare.py
```

The SPARC data files used are public measurement tables from the SPARC page [7].

## References


[1] B. Mueller, A. Osika, K. Xue, and P. Nguyen, *Recovering Relativity and the Standard Model from Observer Overlap Consistency*, OPH compact paper, release r1411, May 3, 2026. https://github.com/FloatingPragma/observer-patch-holography/blob/main/paper/recovering_relativity_and_standard_model_structure_from_observer_overlap_consistency_compact.pdf

[2] B. Mueller, A. Osika, K. Xue, P. Nguyen, M. Poneder, and K. A. Anirudha, *Observers Are All You Need*, OPH synthesis paper, release r1411, May 3, 2026. https://github.com/FloatingPragma/observer-patch-holography/blob/main/paper/observers_are_all_you_need.pdf

[3] B. Mueller and K. Xue, *Federated Echosahedral Screen Microphysics: Patch Hardware, Records, and Observer Synchronization in OPH*, OPH microphysics paper. https://github.com/FloatingPragma/observer-patch-holography/blob/main/paper/screen_microphysics_and_observer_synchronization.pdf

[4] B. Mueller, A. Osika, K. Xue, and M. Poneder, *Deriving the Particle Zoo from Observer Consistency*, OPH particle paper, release r1411, May 3, 2026. https://github.com/FloatingPragma/observer-patch-holography/blob/main/paper/deriving_the_particle_zoo_from_observer_consistency.pdf

[5] Planck Collaboration, *Planck 2018 results. VI. Cosmological parameters*, arXiv:1807.06209. https://arxiv.org/abs/1807.06209

[6] F. Lelli, S. S. McGaugh, and J. M. Schombert, *SPARC: Mass Models for 175 Disk Galaxies with Spitzer Photometry and Accurate Rotation Curves*, Astron. J. 152, 157, 2016, arXiv:1606.09251. https://arxiv.org/abs/1606.09251

[7] F. Lelli, S. S. McGaugh, and J. M. Schombert, SPARC public data page. https://astroweb.cwru.edu/SPARC/

[8] S. S. McGaugh, F. Lelli, and J. M. Schombert, *The Radial Acceleration Relation in Rotationally Supported Galaxies*, Phys. Rev. Lett. 117, 201101, 2016, arXiv:1609.05917. https://arxiv.org/abs/1609.05917

[9] P. Li, F. Lelli, S. S. McGaugh, and J. M. Schombert, *Fitting the radial acceleration relation to individual SPARC galaxies*, Astron. Astrophys. 615, A3, 2018, arXiv:1803.00022. https://arxiv.org/abs/1803.00022

[10] M. Milgrom, *A modification of the Newtonian dynamics as a possible alternative to the hidden mass hypothesis*, Astrophys. J. 270, 365, 1983. https://ui.adsabs.harvard.edu/abs/1983ApJ...270..365M/abstract

[11] J. D. Bekenstein, *Relativistic gravitation theory for the modified Newtonian dynamics paradigm*, Phys. Rev. D 70, 083509, 2004. https://doi.org/10.1103/PhysRevD.70.083509

[12] D. Clowe, M. Bradac, A. H. Gonzalez, M. Markevitch, S. W. Randall, C. Jones, and D. Zaritsky, *A direct empirical proof of the existence of dark matter*, Astrophys. J. Lett. 648, L109, 2006, arXiv:astro-ph/0608407. https://arxiv.org/abs/astro-ph/0608407