Finite Observer Consensus as a Reconstruction Principle: Normal Forms, Gauge Structure, and a Route to the Einstein Field Equation

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Abstract

We ask whether consensus among finite self-reading observer patches can function as a reconstruction principle without identifying finite consistency with physical realization. Each patch has local state, a boundary, durable records, overlap readback, and authorized repair moves. We prove an observer-indexed normal-form theorem, stability bounds under refinement, and complexity classifications for succinct boundary-extension problems. For an assumed oriented twelve-port icosahedral carrier, quadratic descent plus an explicit local-diamond condition yields a schedule-independent public record. A separate classification shows that any compact Lie bracket on the associated twelve-dimensional module with inner action of the alternating group A_5 has Lie algebra $\mathfrak{u}(1) \oplus \mathfrak{su}(2) \oplus \mathfrak{su}(3)$; an explicitly chosen four-band response map realizes such a bracket. Adding an explicitly chosen trace-balanced $3 \oplus 2$ module recovers the familiar anomaly-free fifteen-state exterior content and $\mathbb{Z}_6$ kernel within a declared scan class. These gauge results are finite and depend on the carrier, response, and matter inputs. The conformal two-sphere supplies a Lorentz-group action as mathematics, while its interpretation as physical observer kinematics requires a realization map. If a common refinement tower realizes a Lorentzian event manifold, conserved stress tensor, null balance, entropy–area relation, continuum control, and coupling scale, null tomography and the Bianchi identity yield the Einstein field equation up to a cosmological constant. The common physical tower and continuum limit are not constructed here. The paper separates finite theorems, certified computations, reconstruction implications, and open physical realization maps.

Keywords: foundations of physics; Observer Patch Holography; finite reconstruction; quantum records; Einstein field equation; gauge structure.

1 Introduction and claim boundary

Quantum probability, Lorentzian kinematics, gravitational dynamics, and gauge structure enter standard theories through distinct mathematical inputs. Observer Patch Holography (OPH) asks which structures follow from a specified finite patch system whose components retain records, compare overlap readbacks, and repair disagreement. The source object is narrower than generic observer consistency: it includes a declared twelve-port carrier, explicit repair hypotheses, a finite algebra-state representation, and the interpretation maps stated with each physical conclusion.

The mathematical answer has two layers. Exact results concern observable normal forms, finite record identities, refinement stability, computational obstructions, and the twelve-port response construction. Physical conclusions require further maps from those finite objects to events, currents, clocks, stress, and scales. The Lorentz statement is a group-theoretic consequence of the assumed spherical support. The Einstein statement has a different logical form: an exact coefficient identity is followed by tensor completion from an explicit null-balance, Ward, Bianchi, and scale contract. The modular, stress, entropy, continuum, and area maps are the proposed route to that contract. No physical conclusion follows until one carrier realizes them jointly.

Table 1 gives the four status descriptions used throughout.

Table 1: Claim-status vocabulary used in the manuscript.

Finite theorem	A deductive result proved from displayed premises on a stated finite domain. An exhaustive computation counts as part of the proof only with a specified domain and a soundness argument.
Certified finite computation	An executable witness whose inputs, controls, and output are fixed by a reproducible certificate; execution alone does not promote its interpretation to a theorem.
Reconstruction implication	An exact implication whose antecedents include displayed geometric or physical premises.
Open realization map	A required map from a finite object to a physical event, current, field, clock, scale, or continuum structure for which no construction is asserted.

The distinction is mathematical: a finite implication and a physical realization of its antecedents are different claims. Machine-readable receipts record that distinction and reject a physical verdict when a required map is absent.

Boundary 1.1. This paper claims no completed derivation of particle masses, no source-only determination of the fine-structure constant, no completed cosmology, and no claim that formal verification establishes physical truth. Quantitative surfaces that consume measured values are labeled diagnostics and are never counted as predictions. Section 11 lists the decisive open gates.

Definition 1.2 (Physical realization map)**.** A physical realization map assigns finite public records, event data, currents, modular generators, and scales to physical records, spacetime events, laboratory currents, clocks, and units. It must preserve the algebraic operations, overlap restrictions, causal order, normalizations, and refinement relations consumed by the conclusion in question. A resemblance of spectra, dimensions, or symmetry names is insufficient.

1.1 Principal contributions

The principal mathematical contribution is an observer-indexed normal-form framework that separates existence, observable determination, schedule independence, stability, and computational cost. It gives a canonical partial normalizer, a sharp two-output stability bound, an accumulated refinement bound, and complexity barriers for succinct boundary problems.

The second principal contribution is the finite twelve-port response construction. An inner-action classification first shows that any compact Lie bracket on the twelve-port A_5 module for which the A_5 action is inner has the Standard Model gauge Lie type. The four-band response map then supplies an explicit bracket with that type. Under the displayed trace-balanced matter contract, it also gives a faithful matter image with $\mathbb{Z}_6$ cover kernel and an anomaly-free fifteen-state exterior module. The source inputs, response representation, four coefficients, and exhaustive finite scan are part of the theorem statements; laboratory-current attachment is not.

The finite record-algebra development packages projection events, state updates, partition expectations, and the Clauser–Horne–Shimony–Holt bound in a machine-checked interface. These identities are established finite-dimensional mathematics. The contribution here is their placement on the completed public-record surface and the exact interface connecting them.

The geometry branch distinguishes observer-velocity kinematics from event geometry. The Einstein field equation is presented as tensor completion from a realized null balance, with every modular, stress, entropy, area, Ward, continuum, and scale requirement listed as part of the open realization contract. Reproducible finite simulations test a source construction against parts of that contract without promoting the finite sample to a continuum spacetime.

Operational quantum reconstructions [10, 11, 12], relational and modular accounts of physical facts and time [22, 23], thermodynamic routes to gravitational dynamics [31, 32], and compact-group reconstruction [37, 38] provide the principal comparison points. Born and Lüders rules, Tsirelson's inequality, conformal–Lorentz isomorphisms, Tannaka reconstruction, and the familiar one-generation exterior representation are used as established ingredients. The originality asserted here is limited to the observer-fiber normalizer with sharp refinement control, the semantic transaction contract yielding the local diamond, the inner-action classification and explicit four-band twelve-port response map, the exhaustive selection result within its declared exterior menu, and their placement beside a typed geometry-to-dynamics implication.

1.2 The three axioms

The model uses the following three mathematical postulates.

A1 (oriented twelve-port observer screen). In plain language, the model assigns every carrier a finite observer screen with local state, readback, records, repair moves, checkpoints, typed overlaps, and an oriented twelve-port boundary. Concisely, for each regulator r there is a typed object

$$\mathfrak{N}_r = (\mathcal{P}_r, \mathcal{A}_r, \mathcal{R}_r, \mathcal{I}_r, \mathcal{U}_r, \mathcal{C}_r, N_r, S_r, b_r)$$

whose carrier boundary $K_{r,i} = (P_{r,i}, E_{r,i}, F_{r,i}, o_{r,i})$ has $|P| = 12$, $|E| = 30$, $|F| = 20$, and the oriented icosahedral incidence relations. The bridge $b_r : N_r \to S_r$ carries a designated oriented two-cycle to the fundamental class of the spherical support, and the typed maps commute with refinement. A1 constrains the finite carrier, federation, operational interfaces, and spherical-support tower. It does not imply semantic agreement, repair termination, confluence, a response law, a gauge algebra, or any laboratory identification [4].

A2 (observer agreement). In plain language, observers assign the same operational meaning to accepted data on their shared boundary. Formally, the interpretation functor

$$\mathcal{J}_r : \mathsf{Data}_r \longrightarrow \mathsf{Meaning}_r$$

is natural under every visible restriction, recharting, seam translation, higher-overlap map, federation map, and refinement map. On an overlap O,

$$\mathcal{J}_O(\mathrm{res}_{P\to O}\, d_P) = \mathcal{J}_O(\tau_{Q\to P}\, \mathrm{res}_{Q\to O}\, d_Q).$$

A2 constrains operational meaning on accepted shared data. It does not imply global state extension, termination, confluence, unique normal forms, or durable records [4].

A3 (maximum randomness under agreement constraints). In plain language, the selected state is least informative relative to the declared reference after every observer-visible constraint has been imposed. At finite regulator r, let $\mathcal{K}_r$ be the nonempty convex set of compatible local state families and let

$$\mathcal{D}_r(\rho\|\tau_r) = \sum_{P\in\mathcal{G}_r} w_{r,P} D(\rho_{r,P}\|\tau_{r,P}), \qquad w_{r,P} > 0.$$

The observer cover is state-determining on $\mathcal{K}_r$, and

$$\rho_r = \arg\min_{\rho\in\mathcal{K}_r} \mathcal{D}_r(\rho\|\tau_r).$$

A3 constrains state selection inside one A1-fixed feasible space. It does not select a field list, repair law, response map, particle multiplicity, or continuum limit [4].

None of the axioms contains a gauge group, a particle list, an event-manifold dimension, or a recovery law. A1 does fix a two-dimensional spherical support and hence the Lorentz kinematic type used below. The results below state which structures follow from these axioms, which require added premises, and which realization maps remain open.

1.3 The theorem chain

The results form branches with different premises:

$$\begin{aligned}
\text{finite carrier and repair} &\longrightarrow \text{normal form and record identities,}\\
\text{twelve-port response and displayed matter block} &\longrightarrow \text{gauge algebra and } \mathbb{Z}_6 \text{ matter image,}\\
\text{oriented spherical support} &\longrightarrow \text{Lorentz observer-velocity kinematics,}\\
\text{realized null balance, Ward/Bianchi, and scale} &\Longrightarrow G_{ab} + \Lambda g_{ab} = 8\pi G_N T_{ab}.
\end{aligned}$$

The first two rows are finite constructions. The third consumes the declared spherical support. The fourth is a reconstruction implication. A common modular–stress–entropy tower is the proposed route to its null-balance antecedent and remains an open physical realization. Sections 2 through 8 develop these branches. Section 10 describes the verification stack. Section 11 states the falsification architecture and the open obligations. Section 12 compares the program with neighboring approaches.

2 The primitive finite architecture

Definition 2.1 (Observer patch)**.** An observer patch is a finite object with local state, an observable boundary map, durable records, readback, and a set of authorized repair moves. A patch sees a fragment of the world: its own state and the boundary data of its authorized overlaps.

Definition 2.2 (Overlap and repair)**.** For patches x_i, x_j with an authorized overlap $e = (i, j)$, both induced boundary records must agree for the pair to hold public data. A repair move changes local state while preserving protected readout. A configuration is a normal form when no authorized repair applies.

The carrier realization used throughout is the twelve-port icosahedral architecture of A1: each carrier's boundary packet is combinatorially the oriented icosahedron boundary (P, E, F, o) with $|P| = 12$, $|E| = 30$, $|F| = 20$; carriers federate through typed seam algebras with coherent triple-overlap cocycles into a nerve carrying a degree-one bridge to the oriented spherical support. This is the declared source architecture. We do not claim a uniqueness theorem selecting it among all possible carriers; where results depend on it, the dependence is displayed.

Two exact features of this architecture enter the later constructions. First, the proper rotation group of the icosahedral packet is the alternating group A_5, with sixty rotations acting on the twelve ports; the real port-coefficient space has the character decomposition

$$P_{12} \cong_{A_5} \mathbf{1} \oplus \mathbf{3} \oplus \mathbf{3}' \oplus \mathbf{5}, \tag{1}$$

an exact multiplicity-free decomposition obtained by standard finite-group character theory [25] and checked by certified projectors. Second, record keeping is integer valued: a write appends $+1$, a retraction appends -1, and readback sums atomic events per port; a conservative repair transfers one unit across a seam whenever the oriented mismatch has magnitude at least two. With $V(N) = \sum_i N_i^2$, each such repair strictly decreases V by $2(d-1)$ for mismatch $d \geq 2$, so repair terminates by a finite theorem. Divisibility of total load by twelve is a necessary consensus condition and is not sufficient by itself. For the certificate's source-generated full-pile packet, an explicit eighteen-move schedule reaches consensus.

3 Observable normal forms and finite consensus

A completed configuration is defined as a fixed point of the repair-and-agreement map:

$$T(\mathfrak{U}) = \mathfrak{U}, \tag{2}$$

where T is the composite repair-and-agreement dynamics. Fixed-point existence, observable determination, and schedule independence are separate questions. The following framework isolates them [5].

3.1 Exact observable fibers

Definition 3.1 (Observable quotient system)**.** An observable quotient system is a tuple

$$\mathfrak{S} = (Q, C, \mathcal{B}, B),$$

where Q is a set of configurations, $C \subseteq Q$ is the consistent subset, $\mathcal{B}$ is a set of protected records, and $B : Q \to \mathcal{B}$ is the record map. For $b \in \mathcal{B}$, write $C_b = C \cap B^{-1}(b)$.

The fiber C_b is unrealizable, reconstructing, or ambiguous according as its cardinality is zero, one, or greater than one. This trichotomy avoids assigning a state to a record that has either no consistent extension or several observationally indistinguishable extensions.

Theorem 3.2 (Canonical observable normalizer)**.** *The following statements are equivalent:*

(i) $B|_C$ is injective;

(ii) there is a unique map $N : Q \to C \sqcup \{\perp\}$ that preserves B on its C-valued outputs, fixes every point of C, is constant on each B-fiber, and returns $\perp$ exactly on fibers with no consistent extension;

(iii) $B|_C : C \to B(C)$ *is a bijection.*

If $B(Q) \subseteq B(C)$*, the map is a total idempotent retraction* $Q \to C$*.*

Proof. Injectivity makes every nonempty C_b a singleton, which defines N and forces all its properties fiber by fiber. Conversely, if $c, c' \in C$ have $B(c) = B(c')$, the fixed-point and fiber clauses give $c = N(c) = N(c') = c'$. The third statement is the restriction of the first to the image $B(C)$. □

This normalizer is defined by observable fibers, independent of a chosen repair schedule. A repair relation implements it only when the relation preserves B, reaches the consistent set, and satisfies an appropriate liveness condition. Confluence from one source does not by itself prove determination across distinct sources with the same protected record.

3.2 Approximate outputs and refinement

Let (Q, d_Q) be finite, let $(\mathcal{B}, d_{\mathcal{B}})$ be metric, and let $C = \Phi^{-1}(0)$ be nonempty for a nonnegative consistency residual Φ. For $t, r \geq 0$, define

$$\eta_\Phi(t) = \max\{\mathrm{dist}_Q(x, C) : \Phi(x) \leq t\}, \tag{3}$$

$$\omega_B(r) = \max\{d_Q(c, c') : c, c' \in C,\ d_{\mathcal{B}}(Bc, Bc') \leq r\}. \tag{4}$$

The first modulus turns residual error into distance from consistency. The second measures how well protected observations distinguish consistent outputs.

Theorem 3.3 (Sharp two-output stability bound)**.** *Suppose* B *is* L_B*-Lipschitz. If*

$$\delta_x, \delta_y, \varepsilon \geq 0, \qquad \Phi(x) \leq \delta_x, \qquad \Phi(y) \leq \delta_y, \qquad d_{\mathcal{B}}(Bx, By) \leq \varepsilon,$$

then

$$\begin{aligned} d_Q(x, y) \leq{}& \eta_\Phi(\delta_x) + \eta_\Phi(\delta_y) \\ &+ \omega_B(\varepsilon + L_B[\eta_\Phi(\delta_x) + \eta_\Phi(\delta_y)]) . \end{aligned} \tag{5}$$

At zero residual the ω_B *term is exact, and the coefficient one on each of the two residual-distance terms is optimal in the class of finite metric systems.*

Proof. Choose nearest consistent points c_x, c_y. The two residual moduli bound the outer legs $x \to c_x$ and $c_y \to y$. Lipschitz continuity bounds $d_{\mathcal{B}}(Bc_x, Bc_y)$ by the argument of ω_B in (5); the inverse-observation modulus bounds the middle leg. The triangle inequality gives the result. Exactness at zero residual follows from the definition of ω_B. To test the first residual coefficient, take the two-point metric $Q = \{c, x\}$ with $C = \{c\}$, $d_Q(c, x) = s$, constant B, $y = c$, $\Phi(c) = 0$, and $\Phi(x) = 1$ at $(\delta_x, \delta_y, \varepsilon) = (1, 0, 0)$. The bound reduces to $s \leq \eta_\Phi(1) = s$, so that coefficient cannot be smaller than one. Exchanging x and y proves the same statement for the second coefficient. □

Let (Q_n, d_n) be nonempty finite metric spaces. For $m \geq j \geq n$, let $\rho_{m,n} : Q_m \to Q_n$ be restriction maps satisfying $\rho_{n,n} = \mathrm{id}_{Q_n}$ and $\rho_{m,n} = \rho_{j,n}\rho_{m,j}$. Let $N_n : Q_n \to Q_n$ be total level normalizers. Set

$$a_j = \max_{q \in Q_{j+1}} d_j(\rho_{j+1,j} N_{j+1} q, N_j \rho_{j+1,j} q), \qquad K_{j,n} = \mathrm{Lip}(\rho_{j,n}).$$

Theorem 3.4 (Accumulated refinement bound). *For $m > n$ and $q \in Q_m$,*

$$d_n(\rho_{m,n} N_m q, N_n \rho_{m,n} q) \le \sum_{j=n}^{m-1} K_{j,n} a_j.$$

In particular, if the restrictions are nonexpansive and $\sum_{j=0}^{\infty} a_j < \infty$, then

$$\sup_{m>n} \max_{q \in Q_m} d_n(\rho_{m,n} N_m q, N_n \rho_{m,n} q) \le \sum_{j=n}^{\infty} a_j \longrightarrow 0 \quad (n \to \infty).$$

This is asymptotic naturality of the normalizers.

Proof. Insert the intermediate points $p_j = \rho_{j,n} N_j \rho_{m,j} q$. Consecutive points differ by at most $K_{j,n} a_j$. Summing the telescoping chain from p_m to p_n gives the bound. □

3.3 Computational boundary

The normalizer can be canonical without being cheap. For a Boolean circuit $F : \{0,1\}^n \to \{0,1\}$, take $C = F^{-1}(1)$ and let B_S project onto a declared coordinate set S.

Theorem 3.5 (Complexity of succinct boundary reconstruction). *For succinct Boolean boundary systems:*

(i) deciding whether a protected record has a consistent extension is NP-complete;

(ii) deciding whether $B_S|_C$ is injective is coNP-complete, even when all but one variable are observed;

(iii) deciding whether every protected collar value has some consistent interior extension, equivalently whether an unrestricted total strong repair exists, is Π_2^P-complete.

Proof. For (i), a consistent extension is a polynomial witness and satisfiability is the case $S = \varnothing$. For (ii), noninjectivity is witnessed by two distinct satisfying assignments with the same projection; unsatisfiability reduces to injectivity after adding one hidden tie-breaking bit and one always-consistent baseline assignment. For (iii), a repair exists exactly when the corresponding circuit relation satisfies $\forall d\, \exists w\, R(w, d) = 1$, the canonical one-alternation quantified Boolean formula problem. □

3.4 Repair realization on the twelve-port carrier

For a prepared batch of support-closed transactions, join τ and σ in the conflict graph when

$$W_\tau \cap (R_\sigma \cup W_\sigma) \neq \varnothing \quad \text{or} \quad W_\sigma \cap (R_\tau \cup W_\tau) \neq \varnothing.$$

The conflict components below are obtained by iterating the following finite closure: compute semantic support to a fixed point inside every aggregate, rebuild the conflict graph on the expanded read and write sets, merge every connected component, and repeat until the partition and all aggregate supports are unchanged.

Proposition 3.6 (Transactional local diamond). *Let an accepted repair be an aggregate transaction with read set R_τ, write set W_τ, a read snapshot, and a snapshot-determined payload. Assume:*

(i) every read set contains the semantic dependency closure of its write set, and every affected acceptance functional is revalidated at commit;

(ii) *the terminal closure above exists, and each of its connected conflict components has one coherent canonical aggregate payload;*

(iii) *a prepared component whose snapshot remains valid survives commits of independent components; and*

(iv) *accepted commits preserve the protected record and strictly decrease the integer descent functional.*

Then every one-step quotient peak

$$t \longleftarrow s \longrightarrow u$$

has a quotient state v with $t \longrightarrow v \longleftarrow u$.

Proof. Two different first steps cannot originate in one conflict component because that component has a unique aggregate payload. They therefore originate in distinct components of the terminal conflict closure. Rebuilding the graph after every aggregate support expansion makes their final read and write sets satisfy

$$W_\tau \cap (R_\sigma \cup W_\sigma) = \varnothing, \qquad W_\sigma \cap (R_\tau \cup W_\tau) = \varnothing.$$

Their writes are disjoint and neither commit changes the other's read snapshot. Semantic dependency closure ensures that neither commit changes an acceptance functional consumed by the other. Revalidation and the surviving component rule therefore make both second commits legal. Since the payloads are snapshot-determined and the writes are disjoint,

$$\mathrm{Apply}_\sigma \, \mathrm{Apply}_\tau(s) = \mathrm{Apply}_\tau \, \mathrm{Apply}_\sigma(s) = v.$$

Protected-record preservation makes this equality valid on the physical quotient. □

Theorem 3.7 (Consensus normal form)**.** *Suppose the accepted repair relation satisfies Proposition 3.6, the exact quadratic descent of Section 2, and repair completeness, meaning that its normal forms are exactly the consistent states. Every maximal asynchronous repair schedule from one initial state then terminates at the same consistent quotient normal form. The induced global repair map is idempotent, fixes consistent states, and preserves the protected record.*

Proof. Quadratic descent makes the accepted relation terminating. Proposition 3.6 makes it locally confluent, so Newman's lemma [21] makes it confluent. A terminating confluent relation has a unique normal form below each source. Repair completeness puts that normal form in the consistent set. Applying the normalizer a second time does nothing, and protected-record preservation holds along every accepted edge. □

Boundary 3.8*.* Atomic commits, disjoint write sets, or termination alone do not imply the local diamond. Countermodels omitting semantic dependency closure, canonical aggregation, or the surviving component rule are retained with the finite receipt. The reference engine exhaustively checks its states, peaks, payload hashes, protected records, and descent comparisons. The theorem remains finite and asserts no continuum limit.

The declared class also has gauge compatibility (repair commutes with local relabeling), idempotence of the completed dynamics, and refinement compatibility where proved. The public object is the protected quotient record, not an arbitrary intermediate configuration.

The architecture sits close to distributed agreement in the tradition of Lamport, Shostak, and Pease [19]: patches play the role of protocol nodes, overlap repair of a quorum vote, and the completed fixed point of the decided state. A component paper of the stack proves safety and liveness for a Byzantine-tolerant reading of repair under explicit quorum and partial-synchrony assumptions [7]; the impossibility boundary of Fischer, Lynch, and Paterson [20] is the reason those assumptions are named.

4 Finite record algebra and quantum identities

Once a compare, write, and verify slice has completed, its accessible events are represented in a finite-dimensional $*$-algebra with a normalized state. This algebra-state representation is an explicit input at this stage. The consensus theorem identifies which record is public; it does not derive the classification of finite-dimensional C^*-algebras or the trace pairing.

Theorem 4.1 (Finite public-record identities). *Let P_E be the projector of a completed public event E and ρ the normalized state of the completed record surface. The event weight and nonzero-weight update*

$$\Pr(E) = \mathrm{Tr}(\rho P_E), \qquad \rho \,|\, E = \frac{P_E\,\rho\,P_E}{\mathrm{Tr}(\rho P_E)} \tag{6}$$

define a normalized probability measure on every orthogonal event partition and a normalized positive state on the post-event corner. For self-adjoint dichotomic observables A_0, A_1 and B_0, B_1 in commuting record subalgebras, with $A_i^2 = B_j^2 = I$, define

$$S_{\mathrm{CHSH}} = \mathrm{Tr}[\rho\{A_0(B_0 + B_1) + A_1(B_0 - B_1)\}]\,.$$

Then

$$|S_{\mathrm{CHSH}}| \le 2\sqrt{2}. \tag{7}$$

Proof. Positivity of ρ and $0 \le P_E \le I$ give $0 \le \mathrm{Tr}(\rho P_E) \le 1$. For a complete orthogonal partition (P_k), linearity and $\sum_k P_k = I$ give $\sum_k \mathrm{Tr}(\rho P_k) = 1$. If the event weight is nonzero, $P_E \rho P_E / \mathrm{Tr}(\rho P_E)$ is positive, normalized, and supported on $P_E \mathcal{A} P_E$.

For $\mathcal{B} = A_0(B_0 + B_1) + A_1(B_0 - B_1)$, the commuting-subalgebra calculation gives

$$\mathcal{B}^2 = 4I - [A_0, A_1][B_0, B_1].$$

The two commutator norms are at most two, so $\|\mathcal{B}^2\| \le 8$ and $\|\mathcal{B}\| \le 2\sqrt{2}$. Taking its expectation in ρ proves the stated bound. □

The first identity is the Born rule on this finite surface [13]; the second is Lüders conditioning [14]; the third is the Tsirelson bound [16] for the Clauser–Horne–Shimony–Holt combination [15] at fixed cutoff. Compatibility structure for commuting events follows the standard finite operational pattern within the same library.

Theorem 4.2 (Two-level partition expectation). *Let $(P_i)_{i=1}^k$ be pairwise orthogonal projectors with $\sum_i P_i = I$. Define*

$$\mathcal{P}(X) = \sum_i P_i X P_i, \qquad \mathcal{A}(X) = \sum_i \frac{\mathrm{Tr}(X P_i)}{\mathrm{Tr}(P_i)} P_i,$$

with a zero projector contributing zero. Then $\mathcal{P}$ and $\mathcal{A}$ are positive, unital, trace-preserving idempotent linear maps. The exact range of $\mathcal{P}$ is the commutant of the partition; the exact range of $\mathcal{A}$ is the commutative span of the P_i; and

$$\mathcal{A}\mathcal{P} = \mathcal{A} = \mathcal{P}\mathcal{A}.$$

The average preserves every partition-event weight. For a partition member of nonzero weight, averaging commutes with Lüders conditioning and both orders give the normalized projector.

Proof. Orthogonality gives $\mathcal{P}^2 = \mathcal{P}$ and $\mathcal{A}^2 = \mathcal{A}$. Each map is a sum of positive corner maps, is unital, and preserves the trace. An operator is fixed by $\mathcal{P}$ exactly when its off-diagonal partition blocks vanish, which is equivalent to commuting with every P_i. An operator is fixed by $\mathcal{A}$ exactly when it is a scalar multiple of P_i on each nonzero block. These range descriptions imply $\mathcal{AP} = \mathcal{A} = \mathcal{PA}$. Finally, $\mathrm{Tr}(\mathcal{A}(X)P_i) = \mathrm{Tr}(XP_i)$; applying this identity to $P_i X P_i$ and normalizing gives the conditioning statement. □

These are standard finite-matrix identities related to trace-preserving conditional expectations [17, 18]. The accompanying formal development contributes a checked arbitrary-partition interface, exact range and uniqueness statements, and adapters from projection events to the state-level Clauser–Horne–Shimony–Holt bound [6]. No priority claim is made for Born probability, Lüders conditioning, pinching, or Tsirelson's inequality.

Boundary 4.3. This is a finite operational record interface. It does not reconstruct an arbitrary quantum state space or an interacting continuum quantum field theory. Its role is narrower: the completed consensus record carries the finite probability, conditioning, expectation, and correlation operations consumed by later branches.

5 Overlap defects and edge-center entropy

Overlap agreement has a second layer beyond equality of scalar records. Transport maps can compose only up to a central multiplier. The resulting defect is a finite gluing invariant; removing triangle defects and obtaining endpoint-only transport are distinct operations.

Theorem 5.1 (Central defect strictification and residual holonomy)**.** *Let N_Σ be the nerve of a finite overlap cover. Suppose $U_{ji} = U_{ij}^{-1}$ and*

$$U_{ij}U_{jk} = z_{ijk}U_{ik}, \qquad z_{ijk} \in Z_\Sigma,$$

where the identified coefficient group Z_Σ is abelian and overlap transport acts trivially on it. Then:

(i) z is a Čech 2-cocycle,

$$z_{jkl}z_{ikl}^{-1}z_{ijl}z_{ijk}^{-1} = 1;$$

(ii) its class $[z] \in \check{H}^2(N_\Sigma, Z_\Sigma)$ is invariant under changes of local frame;

(iii) a central edge rephasing removes every triangle multiplier exactly when $[z] = 0$;

(iv) after such a strictification, endpoint-only transport on a charge block $\mathcal{C}_\alpha$ holds exactly when the represented holonomy of every closed loop is the identity on $\mathcal{C}_\alpha$.

Proof. Associativity compares $(U_{ij}U_{jk})U_{kl}$ with $U_{ij}(U_{jk}U_{kl})$ and gives the cocycle identity. A local frame change conjugates the multiplier, hence leaves a central element unchanged. An edge rephasing by a 1-cochain changes z by its Čech coboundary, proving the third statement. Once the edge maps form a strict 1-cocycle, the discrepancy between two paths with common endpoints is the holonomy of the closed loop obtained by composing one path with the inverse of the other. □

The theorem gives a precise finite meaning to gluing curvature. A vanishing triangle class does not imply trivial transport around noncontractible loops. If the center varies as a local coefficient system, the cocycle equation must use the transported, twisted Čech differential; that case lies outside the untwisted statement above.

5.1 Finite edge-center decomposition

Let a regulated collar be cut into left and right halves with common interface Σ. Suppose finite-dimensional Hilbert spaces $\widetilde{H}_L, \widetilde{H}_R$ carry diagonal unitary actions of a finite compact group G_Σ. Write

$$\widetilde{H}_L = \bigoplus_\alpha V_\alpha \otimes H_{L,\alpha}, \qquad \widetilde{H}_R = \bigoplus_\beta V_\beta^* \otimes H_{R,\beta}.$$

Here the V_α are pairwise inequivalent irreducible unitary representations of G_Σ and $d_\alpha = \dim V_\alpha$.

Theorem 5.2 (Invariant collar decomposition and one-sided edge entropy)**.** *The invariant collar space and its sector-preserving algebra have the exact forms*

$$H_C = (\widetilde{H}_L \otimes \widetilde{H}_R)^{G_\Sigma} \cong \bigoplus_\alpha H_{L,\alpha} \otimes H_{R,\alpha},$$

$$\mathcal{A}_C = \bigoplus_\alpha \mathcal{B}(H_{L,\alpha}) \otimes \mathcal{B}(H_{R,\alpha}), \qquad Z(\mathcal{A}_C) = \bigoplus_\alpha \mathbb{C}\, P_\alpha^C.$$

Let $\Omega_\alpha \in V_\alpha \otimes V_\alpha^*$ be the normalized invariant vector corresponding to $d_\alpha^{-1/2} I_{V_\alpha}$. A sector-preserving collar state has the form

$$\rho_C = \bigoplus_\alpha p_\alpha \rho_\alpha, \qquad \rho_\alpha \in \mathcal{B}(H_{L,\alpha} \otimes H_{R,\alpha}),$$

where $p_\alpha \geq 0$, $\sum_\alpha p_\alpha = 1$, and each block with $p_\alpha > 0$ satisfies $\rho_\alpha \geq 0$ and $\operatorname{Tr} \rho_\alpha = 1$. Embed it in $\widetilde{H}_L \otimes \widetilde{H}_R$ through Ω_α and trace out $\widetilde{H}_R$. If $\rho_{L,\alpha} = \operatorname{Tr}_{H_{R,\alpha}} \rho_\alpha$, the resulting left state is

$$\rho_L = \bigoplus_\alpha p_\alpha \frac{I_{V_\alpha}}{d_\alpha} \otimes \rho_{L,\alpha},$$

and its entropy splits as

$$S(\rho_L) = H(p) + \sum_\alpha p_\alpha S(\rho_{L,\alpha}) + \sum_\alpha p_\alpha \log d_\alpha.$$

The edge term is the expectation in ρ_L of the sector observable

$$Z_L = \sum_\alpha (\log d_\alpha) P_\alpha^L,$$

where P_α^L projects onto $V_\alpha \otimes H_{L,\alpha} \subset \widetilde{H}_L$.

Proof. The tensor product of the two representation decompositions contains $V_\alpha \otimes V_\beta^*$. Schur's lemma gives a one-dimensional invariant space when $\alpha = \beta$ and zero otherwise, yielding the invariant direct sum and the center of its sector-preserving algebra. The partial trace of $|\Omega_\alpha\rangle\langle\Omega_\alpha|$ over V_α^* is I_{V_α}/d_α. The displayed expression for ρ_L follows. Entropy of a block-diagonal state is the Shannon entropy of the block weights plus the mean block entropy. Tensor-product additivity contributes $\log d_\alpha$ from each maximally mixed representation factor, and $\operatorname{Tr}(\rho_L Z_L) = \sum_\alpha p_\alpha \log d_\alpha$. ☐

The theorem distinguishes the gauge-invariant collar state from its one-sided reduced state; the representation-dimension term belongs to the latter. Reading Z_L as an area observable requires a physical realization map. Exact quantum Markovity alone also does not identify an arbitrary state-dependent Markov decomposition with these preselected edge-center factors; alignment of the state with the collar decomposition is an additional premise. Related edge-mode and algebra-center decompositions occur in lattice gauge theory [34, 35].

6 From observer velocities to events and spacetime

6.1 Exact kinematics of the spherical support

The oriented conformal spherical support carries

$$\mathrm{Conf}^+(S^2) \;\cong\; \mathrm{PSL}(2,\mathbb{C}) \;\cong\; \mathrm{SO}^+(3,1), \qquad H^3 \;\cong\; \frac{\mathrm{SO}^+(3,1)}{\mathrm{SO}(3)}, \qquad \dim H^3 = 3. \tag{8}$$

These are classical isomorphisms of conformal geometry, consumed as exact mathematics [24]. They identify the three-dimensional hyperbolic homogeneous space of future unit timelike directions. Interpreting those directions as physical observer velocities requires realized events, clocks, and local frames.

Boundary 6.1. Observer-velocity geometry and populated event geometry are distinct objects, and the program keeps them distinct. A populated four-dimensional event base requires record separation, local charts, an open-image condition, affine transition data, a quadratic cone, and causal reachability; these are required constructions, not corollaries of (8). The velocity space H^3 and event localization are typed as separate constructions, and no inference passes from one to the other without a certificate.

Theorem 6.2 (Sufficient gluing criterion for a record-germ event manifold)**.** *Let D be the algebraic direct limit of a cofinal sequence of finite record-germ stages. Suppose the refinement maps make the common-refinement distance $d : D \times D \to [0, \infty)$ a well-defined pseudometric, independent of the chosen stage representatives. Quotient D by zero distance and take its metric completion; call the result X. Then X is a separable, second-countable metric space. Suppose the same cofinal sequence supplies:*

(E1) population of every point of X by realized record germs;

(E2) a covering family of bi-Lipschitz homeomorphisms $\phi_i : U_i \to V_i$ from open subsets $U_i \subset X$ onto open subsets $V_i \subset \mathbb{R}^4$;

(E3) $C^{1,1}$ affine overlap maps satisfying the triple-overlap cocycle;

(E4) $C^{1,1}$ tetrads and inverse tetrads on those charts;

(E5) compatibility on overlaps: the chart metrics induced by the tetrads are related by pullback under the transition maps;

(E6) a nondegenerate quadratic event form of inertia $(1, 3)$, with compatible orientation and time orientation on every overlap; and

(E7) local semantic causal reachability agreeing with the selected cone.

Then the event base is a time-oriented Lorentzian four-manifold of regularity $C^{1,1}$. The future-unit timelike vectors, equivalently observer velocities modulo spatial rotations, form an H^3 fiber over each event. Stable causality and global hyperbolicity are additional conditions.

Proof. The algebraic direct limit of countably many finite stages is countable and dense in its metric completion, so X is separable. Every separable metric space is second-countable and Hausdorff. The covering bi-Lipschitz charts and their open images give the local Euclidean topology, while the affine cocycle gives a consistent $C^{1,1}$ atlas. The tetrads define chartwise Lorentz metrics, and the pullback clause makes them one global $C^{1,1}$ metric. Inertia, orientation, time orientation, and cone-compatible reachability supply the asserted local causal structure. The unit future timelike vectors of each Lorentzian tangent space form $\mathrm{SO}^+(3,1)/\mathrm{SO}(3) \cong H^3$. ∎

6.2 A measured finite Lorentzian signature

The simulation branch measures the event form. Repair dynamics on federated carriers emits semantic events with ancestry; a declared chart combines one ancestry-depth coordinate with three spectral coordinates; a quadratic event form is fitted on a declared training half of the event pairs and evaluated on the held-out half. The signature is therefore held out from the update rule and is classified as a certified finite computation. The cone margin is the minimum held-out signed score, with sign $+1$ for ancestry-comparable pairs and -1 for incomparable pairs. A negative margin means that at least one held-out pair lies on the wrong side of the fitted quadratic cone.

On the reported support-adjusted comparison path with (carrier count, observer count, support width) equal to $(16{,}384, 128, 96)$, $(65{,}536, 256, 96)$, and $(262{,}144, 512, 384)$, the held-out event form has inertia $(1, 3)$ at every rung, with cone margins

$$-5.6, \qquad -3.2, \qquad -1.4 \tag{9}$$

whose magnitudes shrink along that path. The rows change support width and cross-edge density, so they do not constitute a convergence sequence. At 262,144 carriers the same-size support-width-96 control has 312 cross-observer edges and inertia $(2, 2)$, while the support-width-384 row has 1,062 edges and inertia $(1, 3)$. In this finite comparison, changing the support and cross-read structure changes the fitted inertia. The comparison does not isolate a unique mechanism or establish a continuum limit [2].

Boundary 6.3. The chart allocates four coordinates by declaration; the instrument tests signature and full rank on that chart and does not derive the number of spacetime dimensions. The negative cone margins, one Euclidean local fit in the finite source domain below, closed neighborhoods in place of open charts, and the absence of a certified refinement limit prevent promotion of any finite object in this section to a continuum Lorentzian manifold. These conditions are recorded machine-readably in the receipts; the continuum attachment is an open realization map.

6.3 A finite source satisfying part of the local-domain contract

One deterministic capture at 16,384 carriers supplies: a finite causal complex of 2,304 events with exact acyclicity and a strict time function; a certified four-column chart, nondegenerate on the sample, whose global held-out form has inertia $(1, 3)$; six closed observer-visibility neighborhoods with exact induced affine transitions, cocycles, orientation, and time orientation; an observer-visible seam complex of 8,662 carriers and 11,816 seams whose 38 triangles are all frustrated under the declared orientation-reversing seam transport, with lift-ambiguity rank 3,117 over $\mathbb{F}_2$; typed scalar, chiral, and gauge sections with a sign-twisted local derivative whose adjoint, kinetic, covariance, gluing, refinement, and boundary identities are verified by exact integer evaluation; and a signed-graph rank theorem giving a zero twisted kernel, hence an exactly positive dimensionless spectral gap for the declared local operator. The capture is a certified finite computation; its rank and positivity statements are finite theorems.

One provenance graph binds the construction, and an isolated rerun reproduces the canonical receipt content byte for byte. The verifier rejects any receipt that promotes this finite domain to a continuum spacetime, a physical clock, or a physical mass scale.

7 Einstein reconstruction

Theorem 7.1 (Null tomography and metric ambiguity)**.** *Let V be four-dimensional with Lorentz form η. Suppose a symmetric form X satisfies $X(k, k) = 0$ for every null vector k. Then $X = \phi\eta$*

for a scalar ϕ. Null-null values therefore determine a symmetric tensor modulo the metric line. In an η-orthonormal basis with $\eta = \mathrm{diag}(-1,1,1,1)$, set $s = \sqrt{3}/3$ and take the nine null vectors

$$\begin{gathered}(1,\pm 1,0,0), \quad (1,0,\pm 1,0), \quad (1,0,0,\pm 1),\\ (1,s,s,s), \quad (1,s,s,-s), \quad (1,s,-s,s).\end{gathered}$$

On the η-trace-free representative, use coordinates

$$x = (X_{00}, X_{01}, X_{02}, X_{03}, X_{11}, X_{12}, X_{13}, X_{22}, X_{23}), \qquad X_{33} = X_{00} - X_{11} - X_{22}.$$

The corresponding nine-charge design reconstructs the quotient

$$\mathrm{Sym}^2(V^*)/\mathbb{R}\eta.$$

Its determinant is $8192/27$. With the supremum norms on coordinates and charges, its exact decoder obeys

$$\|x\|_\infty \le (2+\sqrt{3})\|q\|_\infty.$$

Proof. Choose coordinates with $\eta = \mathrm{diag}(-1,1,1,1)$ and write every future null ray as $(1,n)$ with $|n| = 1$. The identity

$$X_{00} + 2X_{0i}n_i + X_{ij}n_in_j = 0$$

holds on the unit sphere. Its odd part gives $X_{0i} = 0$. The degree-two spherical-harmonic part gives $X_{ij} = \phi\delta_{ij}$, and the constant part gives $X_{00} = -\phi$. Thus $X = \phi\eta$. The quotient therefore has dimension $10 - 1 = 9$. Substitution of the nine displayed algebraic directions into the stated trace-free coordinate basis gives determinant $8192/27$. The explicit decoder has maximum absolute row sum $2 + \sqrt{3}$, which gives the supremum-norm bound. These two constants are exact analytic calculations. The accompanying Lean development proves that the nine directions are null and formalizes the design map, an explicit left-inverse decoder, injectivity, and the metric-line ambiguity [8]. $\square$

The four-dimensional small-ball coefficients used in entanglement-equilibrium arguments [32] can be isolated from their physical interpretation.

Proposition 7.2 (Small-ball coefficient identity)**.** *Let $G_N > 0$ and $\ell > 0$. If scalars $\delta S_{\mathrm{bulk}}, \delta A, t, f$ satisfy*

$$\delta S_{\mathrm{bulk}} = \frac{8\pi^2\ell^4}{15}\, t, \qquad \delta A = -\frac{4\pi\ell^4}{15}\, f, \qquad \delta S_{\mathrm{bulk}} + \frac{\delta A}{4G_N} = 0,$$

then

$$f = 8\pi G_N t.$$

Proof. Substitution gives

$$\frac{4\pi\ell^4}{15}\left(2\pi t - \frac{f}{4G_N}\right) = 0.$$

The prefactor and G_N are nonzero, so cancellation gives the result. $\square$

The tensor completion begins at an explicitly realized null-balance relation. This is the point at which the physical bridge enters the mathematical implication.

Theorem 7.3 (Tensor completion of a realized null balance)**.** *Let (M, g) be a connected, time-oriented, C^3 Lorentzian four-manifold with Einstein tensor $G^{(g)}_{ab}$. Let T_{ab} be a symmetric C^1 tensor satisfying the independent Ward premise $\nabla^a T_{ab} = 0$. Suppose a common realized tower supplies a constant $\kappa > 0$ such that, at every point and for every null vector k,*

$$G^{(g)}_{ab} k^a k^b = \kappa T_{ab} k^a k^b, \tag{10}$$

and suppose $G_N > 0$ with $\kappa = 8\pi G_N$. Then there is a constant Λ such that

$$G^{(g)}_{ab} + \Lambda g_{ab} = 8\pi G_N T_{ab} \tag{11}$$

on M. If a reference event p_0 and scalar Λ_0 satisfy

$$(G^{(g)}_{ab} - 8\pi G_N T_{ab})\big|_{p_0} = -\Lambda_0 g_{ab}\big|_{p_0},$$

then $\Lambda = \Lambda_0$.

Proof. At each point, Theorem 7.1 applied to $G^{(g)} - \kappa T$ gives a scalar ϕ with $G^{(g)}_{ab} - \kappa T_{ab} = \phi g_{ab}$. The contracted Bianchi identity $\nabla^a G^{(g)}_{ab} = 0$, the Ward premise, and metric compatibility give $\nabla_b \phi = 0$. Connectedness makes ϕ constant. Set $\Lambda = -\phi$ and use $\kappa = 8\pi G_N$. Evaluation at p_0 gives $\Lambda = \Lambda_0$ under the final calibration premise. □

For OPH to supply the null-balance premise (10), one common refinement tower must realize all of the following:

(G1) the event manifold of Theorem 6.2, with sufficient regularity for its Levi–Civita connection and contracted Bianchi identity;

(G2) geometrically normalized cap modular flow and half-sided modular inclusions whose directional charges define one symmetric tensor T_{ab};

(G3) the Ward identity for that tensor, independently of its reconstruction from directional charges;

(G4) the edge-center split of Theorem 5.2, the physical normalization $\delta\langle Z_L\rangle = \delta A/(4G_N)$, the modular first law, and generalized-entropy stationarity;

(G5) one family with $\ell_r > 0$, $\ell_r \to 0$, all named remainders $o(\ell_r^4)$, the continuum diamond-kernel and fixed-volume area formulas used in Proposition 7.2, and enough local observer directions to establish (10); and

(G6) universal coupling of the stress and entropy branches, a vacuum reference, and a physical scale calibration establishing $\kappa = 8\pi G_N$.

The exact finite layer contains null tomography, the edge-entropy split, and the coefficient identity. The joint physical tower, its continuum formulas, Ward identity, area normalization, clock and stress identifications, and scale readings remain open realization maps. The machine-checked composition proves the tensor step from an explicit null-balance premise and separately checks the small-ball arithmetic; it does not construct the bridge between them. The finite signature measurement in Section 6.2 tests one geometric ingredient and does not replace these premises.

8 Two gauge reconstructions and a finite matter image

8.1 Sector reconstruction

The overlap and edge-center branch gives a structural gauge result before a particular compact group is identified.

Theorem 8.1 (Bosonic sector reconstruction)**.** *Suppose a cofinal refinement tail carries zero-obstruction, trivial-holonomy bosonic edge sectors and compatible fully faithful pullback functors. Suppose their closure* Sect_∞ *is an essentially small, additive, idempotent-complete, semisimple rigid symmetric* C^**-tensor category with simple unit and finite-dimensional Hom spaces. Suppose it has a faithful, unitary, strong symmetric monoidal,* $*$*-preserving fiber functor*

$$\mathcal{F} : \mathsf{Sect}_\infty \longrightarrow \mathsf{Hilb}_{\mathrm{fd}}.$$

Then

$$G_{\mathrm{Tan}} = \mathrm{Aut}_\otimes^{u,*}(\mathcal{F})$$

is compact in the topology of pointwise operator convergence and $\mathsf{Sect}_\infty \simeq \mathrm{Rep}(G_{\mathrm{Tan}})$ *as symmetric* C^**-tensor categories. The reconstructed group is unique up to isomorphism for the specified category and fiber functor.*

Proof. Here $\mathrm{Aut}_\otimes^{u,*}(\mathcal{F})$ denotes the unitary monoidal natural $*$-automorphisms of $\mathcal{F}$. For every object X, such an automorphism has a component in the compact group $U(\mathcal{F}(X))$. Naturality, tensor compatibility, symmetry, and $*$-compatibility are closed equations, so this automorphism group is a closed subgroup of the product $\prod_X U(\mathcal{F}(X))$ over a small skeleton and is compact in the subspace topology. The Doplicher–Roberts/Tannaka reconstruction theorem then gives the equivalence [37, 38]. A second compact group compatible with the same fiber functor is identified with the same automorphism group. $\square$

Theorem 8.1 classifies the group encoded by the realized sector data. A trivial sector category reconstructs a trivial group. It does not select the Standard Model group without a suitable sector witness, and it does not identify its group with the finite response group below.

8.2 The twelve-port response algebra

Let A be the icosahedral adjacency operator and J the antipodal involution on the twelve ports. Exact incidence gives

$$10J = A^3 - 4A^2 - 5A + 10I. \tag{12}$$

Put $\varphi = (1+\sqrt{5})/2$. Choose from each antipodal axis one of the certificate's exact coordinate vectors $u_i \in \mathbb{Q}(\sqrt{5})^3$, each with $\|u_i\|^2 = 2+\varphi$, and put $U = [u_1 \ \cdots \ u_6]$. For a real port field f, write

$$b_i = \frac{f_i + f_{Ji}}{2}, \qquad d_i = \frac{f_i - f_{Ji}}{2}, \qquad c = \frac{1}{6}\sum_i b_i, \qquad b^0 = b - c\mathbf{1}.$$

The even coordinates split as $\mathbf{1} \oplus \mathbf{5}$. The odd coordinates split as the rank-three frame channel detected by U and its Galois-conjugate companion detected by $\sigma(U)$, where $\sigma(\sqrt{5}) = -\sqrt{5}$, giving the source-module decomposition

$$P_{12} \cong_{A_5} \mathbf{1} \oplus \mathbf{5} \oplus \mathbf{3} \oplus \mathbf{3}'.$$

Theorem 8.2 (Inner-action closure on the twelve-port module)**.** *Let $\mathfrak{g}$ be a twelve-dimensional compact real Lie algebra and let*

$$\rho : A_5 \longrightarrow \mathrm{Int}(\mathfrak{g})$$

make its underlying real A_5-module $\mathbf{1} \oplus \mathbf{5} \oplus \mathbf{3} \oplus \mathbf{3}'$. Then

$$\mathfrak{g} \cong \mathfrak{u}(1) \oplus \mathfrak{su}(3) \oplus \mathfrak{su}(2).$$

The center is the unique trivial line. Up to exchanging $\mathbf{3}$ and $\mathbf{3}'$ by the outer automorphism of A_5, the $\mathfrak{su}(2)$ ideal carries $\mathbf{3}$ and the $\mathfrak{su}(3)$ ideal carries $\mathbf{3}' \oplus \mathbf{5}$.

Proof. Compactness gives the reductive decomposition $\mathfrak{g} = \mathfrak{z} \oplus [\mathfrak{g}, \mathfrak{g}]$. Inner automorphisms fix the center pointwise. The displayed A_5-module has a one-dimensional fixed space, so $\dim \mathfrak{z} \leq 1$.

If $\mathfrak{z} = 0$, the low-dimensional classification of compact simple Lie algebras forces a twelve-dimensional compact semisimple algebra to be $\mathfrak{su}(2)^4$ [36]. Inner automorphisms preserve each simple ideal. On each three-dimensional ideal, the restriction of the A_5 action is either trivial or faithful because A_5 is simple. Its fixed-space dimension is therefore three or zero: a faithful A_5 subgroup of SO(3) cannot fix an axis, since a finite rotation group fixing an axis is cyclic. The fixed-space dimension of $\mathfrak{g}$ would be a multiple of three, contradicting the one-dimensional fixed line. Hence $\dim \mathfrak{z} = 1$.

The semisimple part has dimension eleven. The same low-dimensional classification leaves only $11 = 8 + 3$, realized by $\mathfrak{su}(3) \oplus \mathfrak{su}(2)$. Adding the center proves the Lie-algebra claim. The center must be the unique fixed line. The three-dimensional simple ideal cannot carry the trivial action, since that would add three fixed directions; hence it carries $\mathbf{3}$ or $\mathbf{3}'$, and the eight-dimensional ideal carries the complementary triplet together with $\mathbf{5}$. □

Boundary 8.3. Theorem 8.2 is a classification theorem. It does not prove that a compact bracket or an inner A_5 action exists on the port module. The four-band map below supplies one such bracket under its displayed response premises.

Let

$$\Phi_0(b^0) = \sum_{i=1}^{6} b_i^0 u_i u_i^{\mathsf{T}}, \qquad \widehat{x}\, y = x \times y.$$

The response construction supplies the space

$$H = \mathbb{C}^3_E \oplus \mathbb{C}^3_W$$

and four nonzero rational coefficients $\lambda_1, \lambda_5, \lambda_3, \lambda_{3'}$, one on each source band. These are explicit branch premises. The two three-dimensional response blocks match the two rank-three odd channels; they are not asserted to be a derived physical Hilbert space. Write v_p for the signed coordinate vector at port p. For $g \in A_5$, let $R_g \in \mathrm{SO}(3)$ be the exact rotation satisfying $R_g v_p = v_{g(p)}$ on the oriented port frame, and define

$$\Pi(g) = \mathrm{diag}(R_g, \sigma(R_g)).$$

The target A_5 action on $\mathfrak{u}(H)$ is conjugation by $\Pi(g)$.

Theorem 8.4 (Four-band finite port-response algebra)**.** *On the declared twelve-port carrier and supplied response representation, define*

$$K(f) = \mathrm{diag}\big(\lambda_3 \widehat{U d} + i\{\lambda_1 c I_3 + \lambda_5 \Phi_0(b^0)\},\\ \lambda_{3'} \widehat{\sigma(U) d}\big) \in \mathfrak{u}(H). \tag{13}$$

Then K is injective, its image is commutator-closed, and

$$K(g\cdot f) = \Pi(g)K(f)\Pi(g)^*.$$

The induced A_5 action on $\operatorname{im} K$ is inner. Let $K^{-1} : \operatorname{im} K \to P_{12}$ denote the inverse of K onto its image. With the bracket pulled back from that image,

$$[x, y]_K = K^{-1}([K(x), K(y)]_{\mathrm{mat}}) \, ,$$

one has

$$(P_{12}, [\,\cdot\,,\cdot\,]_K) \cong \mathfrak{u}(3) \oplus \mathfrak{so}(3) \cong \mathfrak{u}(1) \oplus \mathfrak{su}(3) \oplus \mathfrak{su}(2). \tag{14}$$

Its center is the uniform one-dimensional port line, its derived algebra has dimension eleven, and the five-dimensional A_5 band is noncentral. The map has a positive-definite invariant Hilbert–Schmidt pullback.

Proof. Equation (12) is obtained by evaluating both sides on the four distinct adjacency eigenspaces $\mathbf{1}, \mathbf{3}, \mathbf{3}', \mathbf{5}$. The source protocol solves the common farthest-shell filter on each port and gives the signed response $-J$.

The constant coordinate maps to $i\mathbb{R}I_3$. The six rank-one axis matrices $u_i u_i^{\mathsf{T}}$ form a basis of the symmetric 3×3 matrices and satisfy

$$\sum_i u_i u_i^{\mathsf{T}} = (5 + \sqrt{5})I_3.$$

Their sum-zero coefficients therefore map isomorphically to the traceless symmetric matrices under Φ_0. The first two even terms span $i\,\mathrm{Sym}_3(\mathbb{R})$. The map U is an isomorphism on the $\mathbf{3}$ odd band and annihilates the $\mathbf{3}'$ band; $\sigma(U)$ has the complementary property. Their hat maps span one copy of $\mathfrak{so}(3)$ in each response block. Because every $\lambda_\bullet$ is nonzero, these four images are independent and K has rank $1 + 5 + 3 + 3 = 12$.

In the first block, $i\,\mathrm{Sym}_3(\mathbb{R}) \oplus \mathfrak{so}(3) = \mathfrak{u}(3)$ as a real Lie algebra, while the second block is $\mathfrak{so}(3)$. Thus the image is exactly $\mathfrak{u}(3) \oplus \mathfrak{so}(3)$ and is closed under commutators. Pullback of the matrix commutator satisfies bilinearity, antisymmetry, and Jacobi and gives (14). The derived algebra is $\mathfrak{su}(3) \oplus \mathfrak{so}(3)$, of dimension $8 + 3 = 11$; the remaining line is the center of $\mathfrak{u}(3)$. Direct evaluation identifies it with the uniform port vector and exhibits a nonzero commutator involving the five-dimensional band. Equivariance, positivity of $-\operatorname{Re}\operatorname{tr}(K(f)K(f'))$ are exact calculations in $\mathbb{Q}(\sqrt{5})$ reproduced by the released certificate. It also verifies, for all sixty elements, that $\Pi(g)$ is the exponential of an element of $\operatorname{im} K$; hence its conjugation action is inner. The finite refinement and persistence diagrams recorded in the certificate are checked there as separate commuting-square computations. □

Boundary 8.5. The A_5-module decomposition alone does not determine a Lie bracket: the same module also carries the zero bracket. The independent inputs to Theorem 8.4 are the carrier incidence and orientation, the supplied response representation, and four nonzero band coefficients. No candidate Lie algebra, gauge group, particle list, or measured coupling enters the definition of K; the Lie-algebra identification is computed after the map is fixed. The theorem constructs the pulled-back bracket for this response map. It does not prove that the response representation or its coefficients are forced by A1–A3, and it contains no uniqueness claim over all A_5-equivariant brackets or all finite carriers.

8.3 Global form and anomaly-free exterior module

Theorem 8.6 (Finite matter image and exhaustive exterior-menu selection)**.** *Let*

$$V = \mathbb{C}^3_{-1/3} \oplus \mathbb{C}^2_{1/2}$$

be an additional, explicitly supplied trace-balanced matter representation of the Lie algebra identified in Theorem 8.4. This $3 \oplus 2$ matter space is distinct from the supplied response space $H = \mathbb{C}^3_E \oplus \mathbb{C}^3_W$ and is not derived from it. Let

$$\widetilde{G} = \mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$$

act on V by

$$(g_3, g_2, z) \cdot (v_3, v_2) = (z^{-2} g_3 v_3, z^3 g_2 v_2).$$

The differential of this action gives the displayed hypercharges. Its exterior module has the exact branching

$$\Lambda^2 V \oplus \Lambda^4 V = Q \oplus u^c \oplus e^c \oplus d^c \oplus L \tag{15}$$

with

component	$\mathrm{SU}(3) \times \mathrm{SU}(2)$ *type*	Y
Q	$(3, 2)$	$1/6$
u^c	$(\bar{3}, 1)$	$-2/3$
e^c	$(1, 1)$	1
d^c	$(\bar{3}, 1)$	$1/3$
L	$(1, 2)$	$-1/2$

This is the familiar one-generation $\mathbf{10} \oplus \overline{\mathbf{5}}$ branching associated with SU(5) *unification [39]. It has total complex dimension fifteen. All perturbative gauge and mixed gravitational anomalies vanish, and the number of weak doublets, counted with color multiplicity, is four. The common kernel of the action of $\widetilde{G}$ on V, and hence on $\Lambda^2 V \oplus \Lambda^4 V$, is $\mathbb{Z}_6$. Hence the maximal faithful matter image is*

$$S(U(3) \times U(2)) \cong \frac{\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)}{\mathbb{Z}_6}. \tag{16}$$

Define

$$G_{\text{packet}} := S(U(3) \times U(2)),$$

*the maximal faithful image of the displayed source-model module. This quotient is familiar as a possible global form of the Standard Model gauge group [41]. The scan menu consists of the ten nontrivial irreducible summands of $\Lambda^\bullet V$ after the invariant vacuum $\Lambda^0 V$ and invariant top line $\Lambda^5 V$ have been excluded by declaration. Among all $2^{10} = 1{,}024$ subsets of this menu, imposing nonemptiness, chirality, and vanishing of the mixed gravitational–*U(1)*,* $\mathrm{SU}(3)^2\mathrm{U}(1)$*,* $\mathrm{SU}(2)^2\mathrm{U}(1)$*, and* $\mathrm{U}(1)^3$ *traces leaves exactly two rank-fifteen masks. They are exchanged by charge conjugation, and the exterior-parity grading is computed as an output.*

Proof. The exterior-sum identity

$$\Lambda^k(A \oplus B) = \bigoplus_{p+q=k} \Lambda^p A \otimes \Lambda^q B$$

gives the displayed five components and charges. The anomaly-cancellation conditions are standard [40]. With the conventional quadratic indices $T(3) = T(2) = 1/2$, the pure color anomaly is

$$\mathrm{SU}(3)^3 : \qquad 2 - 1 - 1 = 0,$$

where the two color fundamentals occur in the weak doublet and the two antifundamentals are u^c and d^c. The local $\mathrm{SU}(2)^3$ anomaly vanishes identically because the doublet is pseudoreal. The mixed anomalies are

$$\mathrm{SU}(3)^2\mathrm{U}(1): \quad \frac{1}{6} - \frac{1}{3} + \frac{1}{6} = 0, \qquad \mathrm{SU}(2)^2\mathrm{U}(1): \quad \frac{1}{4} - \frac{1}{4} = 0.$$

The abelian cubic and mixed gravitational sums are

$$6\Big(\frac{1}{6}\Big)^3 + 3\Big(-\frac{2}{3}\Big)^3 + 1 + 3\Big(\frac{1}{3}\Big)^3 + 2\Big(-\frac{1}{2}\Big)^3 = 0,$$

$$6\Big(\frac{1}{6}\Big) + 3\Big(-\frac{2}{3}\Big) + 1 + 3\Big(\frac{1}{3}\Big) + 2\Big(-\frac{1}{2}\Big) = 0.$$

There are three color copies of the Q doublet and one L doublet, so the Witten parity is even [42].

For the displayed cover action, the element

$$\Big(e^{2\pi i/3} I_3, -I_2, e^{i\pi/3}\Big)$$

acts trivially on every row of the table and generates a cyclic group of order six. Direct center-action enumeration proves that no larger common kernel exists; the independent integer presentation has Smith invariants $(1,1,1,1,1,6)$. The cover action has determinant one on V, so $\Lambda^4 V \cong V^*$ equivariantly. Its kernel on the displayed exterior module therefore equals its kernel on V. This gives (16). Finally, the finite scan evaluates nonemptiness, chirality, and the four displayed scan traces on all 1,024 masks in the declared ten-summand menu. Its only survivors are the two exterior-parity masks; the explicit conjugation permutation exchanges them. Parity is recorded after selection and is not a selection predicate. □

Boundary 8.7. The finite response group G_{packet} and the sector-reconstructed group G_{Tan} arise from different data. Their equality requires a commuting-square identification that has not been constructed. Laboratory current attachment, physical matter identification, exclusion of extra light sectors, scalar attachment, family multiplicity, Yukawa couplings, mixing, masses, and a continuum quantum-field realization remain open realization maps. The theorem classifies a finite source-model representation and its maximal faithful image. The exclusion of $\Lambda^0 V$, $\Lambda^5 V$, direct sums, vectorlike additions, and neutral singlets is not derived by this scan.

9 Physical interpretation maps and their consequences

The preceding theorems concern finite records, explicit response maps, and reconstruction implications. They become claims about nature only when a physical carrier realizes the operations and identifications in Definition 1.2. Table 2 records the resulting implications and the premise that can fail.

Table 2: Physical questions, reconstruction consequences, and unrealized premises.

Question	**Consequence of the stated realization**	**Unrealized premise**
When is a measurement public?	A completed record is a schedule-independent normal form; on the declared algebra-state surface its event projectors obey Theorem 4.1.	A laboratory system must instantiate durable records, readback, protected boundaries, and the algebra-state map.
When is event geometry Lorentzian?	A common record-germ tower satisfying (E1)–(E7) gives the Lorentzian four-manifold of Theorem 6.2; the finite instrument separately measures inertia $(1,3)$ on its declared path.	A physical refinement family must supply the common-refinement pseudometric, population, covering charts, compatible tetrads, cone, and causal premises.
What equation governs the gravitational response?	A realized null balance, Ward conservation, the Bianchi identity, and an independent scale identification give the Einstein field equation of Theorem 7.3.	The common physical tower, continuum control, clock, stress, area, vacuum, and scale maps remain open.
How do the gravitational and gauge branches share a source?	One carrier can feed the event/modular branch and the twelve-port response branch, yielding both conclusions from a common source.	Joint physical realization and equality of G_{packet} with G_{Tan} have not been established.
Why this gauge type and matter image?	On the supplied inner-action response and displayed matter representation, Theorems 8.2–8.6 give the compact Lie type, $\mathbb{Z}_6$ quotient, and anomaly-free rank-fifteen finite module.	Laboratory currents, quantum fields, physical matter poles, family multiplicity, and masses require further maps.
Why could a string description be effective?	If persistent one-dimensional seam or flux defects are schedule-independent or gauge-equivalent and admit scale separation plus a controlled local worldsheet effective action with finite tension, a derivative expansion, and compatible splitting and joining, they supply a candidate effective-string description.	No such defect sector, gauge-equivalence theorem, scale separation, worldsheet action or measure, error control, Weyl-anomaly treatment, spectrum, coupling map, or duality is constructed.

9.1 Gravity through the modular-entropy map

OPH begins with finite observer patches instead of a metric field. Each patch has local state, a boundary, protected records, readback, and repair moves. Consensus supplies public record classes. A record-germ realization satisfying (E1)–(E7) supplies the event manifold and its causal cone. On that same refinement tower, modular flow supplies a normalized local time generator, half-sided inclusions supply positive null translations, and null tomography assembles their charges into local stress. The edge-center entropy split, area normalization, shrinking-family control, and fixed-volume stationarity would then have to establish the null-balance premise. Null tomography, Ward conservation, the Bianchi identity, and the scale map complete that premise to the Einstein field equation in Theorem 7.3.

This is a route with a visible break point. The normal-form, entropy, and tomography state-

ments have exact finite layers. The passage to gravity starts only when one physical tower realizes the event, modular, continuum, universal-coupling, vacuum, and scale premises together.
Intuition. Under the stated realization map, the gravitational response represents the large-scale compatibility condition associated with repaired records. Local observers extend and compare records; the metric describes the stable causal relation among those records. Modular flow describes how a patch reads change, and entropy stationarity fixes the response needed for the local accounts to remain compatible.

9.2 A common source for gravitational and gauge branches

In this framework, the candidate fundamental theory that combines the gravitational and gauge sectors is one observer-consensus tower with two readout branches. The overlap, record, and repair structure feeds the event and modular branch. The twelve-port incidence, response map, and trace-balanced blocks feed the gauge branch. A single self-reading carrier could therefore supply both sets of antecedents.

This common source neither identifies gravity with a gauge boson nor fixes all couplings. It also leaves two gauge constructions distinct: G_{Tan} is reconstructed from a sector category, while G_{packet} is the maximal faithful image of the explicit finite response and matter module. A completed unification requires joint physical realization and a proof that the corresponding current diagrams commute.
Intuition. One object performs two tasks. Its overlap structure organizes events and the gravitational response; its boundary response organizes charges. The proposed architecture takes the carrier and its self-reading dynamics as their common source, with spacetime and gauge structure appearing as distinct public readouts.

9.3 Completed records as measurement events

The finite construction locates a measurement at the transition from a private, repairable record to a protected public normal form. Event projectors, weights, and state updates are then defined on the completed algebra-state surface. A physical application must identify actual durable records and verify the transaction and readback premises. The theorem does not construct the state space of an interacting continuum field theory.
Intuition. Within the model, a record is called public when it has survived every authorized comparison and no repair can change it. Probability describes the completed record surface; state update moves from one public record class to the next. The construction locates observation in the bookkeeping that patches perform on their shared boundaries.

9.4 Event geometry from record germs

The spherical support identifies the Lorentz group and the hyperbolic space of timelike directions without supplying physical events. Events enter only through the direct limit of separating record germs. Theorem 6.2 states a sufficient gluing criterion: four-component charts with open images, compatible affine transitions, tetrads, a Lorentzian quadratic form, and causal reachability must all occur on one populated refinement limit. The criterion packages these inputs into one Lorentzian manifold; it does not select four dimensions or Lorentz signature from weaker data. The finite instrument measures rank and inertia on a prescribed chart, while its negative cone margins and absent cofinal limit prevent continuum promotion.
Intuition. Record germs say when two finite histories describe the same event and how nearby events can be distinguished. A spacetime appears only when those local descriptions cover the completed

event set, agree on overlaps, and carry one compatible causal cone. The manifold records the stable way in which public events fit together.

9.5 Gauge type and the finite matter image

The twelve-port coefficients split under the alternating-group action into bands of dimensions 1, 3, 3, and 5. If a compact Lie bracket carries that action by inner automorphisms, Theorem 8.2 forces one central direction together with simple ideals of dimensions three and eight. The response map K realizes this algebra explicitly. The matter space V is a separate supplied representation; its exterior algebra gives the familiar fifteen-state branching, and the exhaustive scan proves the two-mask selection only inside its declared ten-summand menu. Physical gauge fields, matter poles, families, and masses require current and continuum realization maps.

Intuition. The carrier exposes twelve response channels. Icosahedral symmetry groups them into four inequivalent bands, while inner compact rotations determine how those bands can close under commutators. The finite response map turns that organization into the Standard Model gauge Lie type. A separate matter module then tests which charges can coexist without anomalies; it does not turn the finite channels into observed particles.

9.6 Why a string description could be effective

String theory is neither an input to the finite construction nor a consequence of the theorems proved here. Persistent one-dimensional seam or flux defects would supply candidate string kinematics only if their histories were schedule-independent or gauge-equivalent and formed a controlled two-dimensional continuum. A usable effective description would also require scale separation, a local reparametrization-invariant action or measure with finite renormalized tension, a controlled derivative expansion and error bounds, closure of the retained modes, and compatible interaction weights for splitting and joining [27, 26]. Under those assumptions, the collective modes could be organized in worldsheet language. Such a description would concern collective extended modes of the observer network. A fundamental perturbative-string interpretation would additionally require quantum Weyl-anomaly cancellation and modular consistency; a noncritical effective-string interpretation would require its own controlled regime. Neither route, nor its spectrum, target-space map, coupling, and relation to the gauge and gravitational branches, is constructed here.

Intuition. Tracing a persistent line defect through a chosen repair history defines a two-dimensional combinatorial history. It becomes a physical worldsheet only if alternative repair histories are gauge-equivalent and a realized clock, target-space map, scale separation, and controlled continuum limit exist. Under those premises, the sheet is a candidate coarse-grained degree of freedom; its vibrations, splitting, and joining can be organized with effective-string variables.

10 Machine verification and reproducibility

Each result maps to an artifact class:

- a Lean library with no admitted propositions in the dependency closure reported here, including formal premise boundaries and countermodels;
- exact-arithmetic code receipts: integer and rational computations, interval certificates with outward rounding, and $\mathbb{F}_2$ rank computations, each emitting a canonical hashed payload;

- simulation receipt bundles pinning the applicable source or module revision, configuration, seed where stochastic, grids, tolerances, and output hashes, with deterministic reserialization distinguished from fresh source replay;
- verifiers that recompute verdicts from clause vectors and reject receipts whose stored verdict disagrees, so a caller cannot assert a truth flag directly;
- adversarial negative controls: every certificate ships with mutations that must be detected, and a control that cannot fail is treated as a defect of the certificate, not as support;
- provenance rules under which machine-readable ledgers classify every quantitative row as source result, reconstruction implication, diagnostic, or rejected candidate, and reject a stronger classification unsupported by the recorded dependencies.

The repository, receipts, schemas, and rebuild instructions are public [1, 2]. The formal and executable stack checks the claims assigned to it and enforces their stated boundaries. Analytic arguments in this manuscript retain their displayed mathematical proofs, and no part of the stack establishes physical realization.

10.1 Artifact map

Table 3 identifies the principal artifacts for each result. Repository continuous integration checks the theorem-count floor, rejects admitted proofs in the public library, and rebuilds the repository-local finite certificates used here. Simulator replay is governed by the simulator receipts and reproduction commands cited below.

Table 3: Principal verification artifacts and their scope.

Result	**Principal artifacts**
Observable normalizer, stability, refinement, and complexity	`Lean/ObservableNormalForms/` and the exact proofs and reductions accompanying `extra/observable_normal_forms.tex`
Transactional diamond and consensus normal form	general local-diamond and Newman arguments proved in this manuscript; `Lean/ObservableNormalForms/ObservableNormalForms/ObserverConfluence.lean` for observation-relative endpoint uniqueness; `code/consensus/verify_issue_517_proof_obligations.py` and its `code/consensus/runs/issue_517_proof_obligations.json` receipt for one finite transaction model and its negative controls
Event algebra, Born, Lüders, Tsirelson	`Lean/EventAlgebra/` (`Basic`, `Lueders`, `Tsirelson`, `ExpectationBound`, partition and state modules); per-module axiom audits
Central defects and edge-center entropy	overlap-cocycle and one-sided reduction proofs in this manuscript; `Lean/ObserverPatchHolography/EinsteinBranch/Entropy.lean` for scalar entropy bookkeeping after the split; `Lean/ObserverPatchHolography/CollarStates.lean` for the identity-channel model and its no-go boundary

Table 3 continued

Result	Principal artifacts
Port action, commutant, coefficient algebra	files under `Lean/Screen/`: `A5PortAction.lean`, `A5Commutant.lean`, `A5IncidenceResponse.lean`, and `Compact12.lean` for the abstract matrix algebra; the exact port map and A_5 covariance are checked by `code/a5_closure/port_current_inner_certificate.py`, `code/a5_closure/manifests/port_current_response_reference.json`, and `code/a5_closure/receipts/port_current_inner_reference.receipt.json`; compact-simple classification remains the analytic input used in Theorem 8.2
Global form, $\mathbb{Z}_6$ kernel, and exterior selection	`Lean/Screen/Z6Exact.lean` for the abstract lattice quotient, `TraceBalancedKernel.lean` for central-parameter arithmetic, and `ExteriorSelection.lean` for the declared finite scan
Record-germ event-manifold gluing	analytic theorem proved in this manuscript; finite chart, transition, and signature receipts test only selected antecedents
Null tomography, small-ball arithmetic, and tensor completion	`Lean/ObserverPatchHolography/EinsteinBranch/Tensor.lean` (null directions, design/decoder, injectivity, and metric ambiguity); `Lean/docs/EINSTEIN_BRANCH_INDEX.md`; analytic determinant and norm calculations in this manuscript; explicit null-balance, Ward, Bianchi, and scale premises
Signature ladder and control	`evidence/einstein_convergence/` (four-row manifest, hashed arrays; pinned simulator revision and replay comparison procedure in its README)
Finite local source domain	staged receipts under `data/local_domain/` and the bundle verifier under `oph_fpe/local_domain/` in the simulator repository

11 Empirical meaning, falsifiability, and open obligations

The program maintains a preregistration architecture: custody-bound target definitions, exclusion thresholds, precision floors, and decision rules fixed before comparison, with failed hypotheses retained in a negative ledger [3]. Those exploratory quantitative programs are outside the mathematical case made in this paper.

The program's empirical case rests on three decisive gates:

1. **Physical realization of the common refinement tower.** The Einstein composition of Theorem 7.3 needs one source-derived tower satisfying its premises jointly; the signature instrument and its controls measure progress toward exactly this object.
2. **A source-only current and continuum realization.** The gauge and matter results of Section 8 await laboratory current attachment and a continuum operator limit; identifiability audits delimit which attachments the registered source interface cannot supply. This gate requires source-visible observables beyond that interface.

3. **One preregistered quantitative forecast not used in architecture selection.** A sealed machine-readable prediction contract fixes the comparison custody. The first source-visible candidate is scored once; a complete negative inventory is an acceptable published outcome.

Each gate can fail, and the second one can fail provably. The program treats a rigorous non-identifiability theorem as a result of equal standing with a positive construction, because it tells the theory where additional observable structure must come from.

12 Discussion and comparison

Operational reconstructions derive quantum structure from information and composition principles [10, 11, 12]. OPH makes a narrower statement at the quantum stage: a completed finite record, once represented by an algebra-state pair, carries the standard event, conditioning, expectation, and correlation identities. The distinctive input is the observer-indexed normal form and its transaction contract. The algebra-state representation remains explicit.

Relational quantum mechanics treats physical facts as relational [22]; the thermal-time program reads time from modular structure [23]. OPH combines these themes by attaching public facts to protected overlap records and consuming normalized modular flow only on a typed common tower. Its local-diamond proof also has a direct distributed-systems neighbor in asynchronous agreement [19, 20].

The gravitational branch belongs to the family of thermodynamic and entanglement-based routes to geometry [31, 32, 33]. Bisognano–Wichmann modular flow and half-sided modular inclusions supply established analytic ingredients [28, 29, 30]. The contribution of Theorem 7.3 is the explicit composition contract: event, modular, stress, entropy, asymptotic, vacuum, and scale premises appear in one statement, while the finite null ambiguity identifies exactly where a metric-proportional term enters.

The gauge section also contains two logically separate routes. Doplicher–Roberts/Tannaka reconstruction supplies the structural sector group [37, 38]. The finite packet group comes from the explicit twelve-port response map and exterior-module scan. The equality of these groups is an open commuting-square problem. This separation prevents the standard categorical theorem from being counted as a derivation of the specific packet group.

For a foundations readership, the methodological claim is also concrete. Finite theorems, certified computations, reconstruction implications, and physical realization maps are different mathematical objects. Keeping those types visible makes every emergence claim auditable and gives each claim a specific failure mode. A3 also places agreement constraints before state selection. The physical adequacy of that ordering depends on the open gates in Section 11.

13 Conclusion

For the specified finite observer-patch architecture, observable fibers give a canonical partial normalizer, stability moduli control approximate and refined outputs, and succinct boundary problems have explicit complexity barriers. On the transactional carrier, semantic dependency closure and revalidation yield the local diamond, so quadratic descent gives a schedule-independent public record. The completed finite algebra-state surface then carries the standard probability, conditioning, expectation, and correlation identities.

The principal finite gauge construction is the twelve-port response map. The inner-action closure theorem first classifies the only possible compact inner-action Lie type on its A_5 module.

For the declared response representation and four nonzero band coefficients, the pulled-back bracket yields $\mathfrak{u}(1) \oplus \mathfrak{su}(2) \oplus \mathfrak{su}(3)$. Trace balance and the exterior module give the global $\mathbb{Z}_6$ quotient and an anomaly-free rank-fifteen representation, while the exhaustive scan isolates its charge-conjugate pair within the stated class.

The geometry and dynamics results have a different status. A Lorentz-group action and its hyperbolic homogeneous space follow from the oriented spherical support; their physical interpretation requires event, clock, and frame maps. A Lorentzian event manifold follows only under the displayed event premises. The Einstein field equation follows only from one common tower satisfying the modular, stress, entropy, asymptotic, vacuum, and scale premises. The physical tower, continuum limit, laboratory-current attachment, quantum-field realization, and remaining interpretation maps are open. The paper therefore offers a finite reconstruction framework with exact results and explicit physical hypotheses; it does not claim a completed derivation of all physics, particle masses, a cosmology, or a proof that the simulator describes our universe.

Reproducibility statement

All theorems, certificates, simulation configurations, receipts, and the formal library are public in the project repositories [1, 2]. The paper release manifest records PDF hashes and sizes. Computational receipt bundles pin the applicable source or module revision, configuration, seed where stochastic, and output hashes. Clean-checkout rebuild instructions and negative controls accompany the artifacts. The claim ledger and prediction register are machine readable [3].

Three compact companion files accompany this submission. Online Resource 1 contains the Lean source tree, pinned toolchain and dependency manifests, and build instructions for the formal results. Online Resource 2 contains the exact Python verifiers, manifests, receipts, negative controls, and tests for the finite response and proof-obligation certificates. Online Resource 3 contains the hash-bound finite signature-ladder and local-domain evidence, their manifests, and replay instructions. The complete simulation source is maintained in the public simulator repository at the evidence-producing revisions [2].

This manuscript synthesizes and strengthens results whose longer proofs also appear in public component preprints on observable normal forms, finite event algebras, consensus, Einstein reconstruction, and gauge structure [5, 6, 7, 8, 9]. The component relationship is disclosed here so that overlap is visible.

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Data availability. The finite witnesses, simulation configurations, receipts, claim ledgers, and release manifests supporting this study are available in the public OPH and simulation repositories [1, 2]. Individual receipts and their bundle documentation provide the source and output provenance for computational claims. Online Resource 3 supplies the finite signature-ladder and local-domain evidence cited in the manuscript.
Code availability. The Lean sources, exact-arithmetic verifiers, finite scans, simulation code, tests, and rebuild instructions are available in the same public repositories [1, 2]. The computational claims in this manuscript identify their principal files in Section 10. Online Resources 1 and 2 supply compact source snapshots for the formal and exact-computation claims.
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