Finite E_8/Spin(8) Triality Certificate for an Alt(9) Double Cover

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July 8, 2026

Paper release: r1519 **Released:** July 8, 2026

Abstract

This note records a compact finite exceptional-symmetry sidecar for OPH. The certificate concerns exact matrix data for E_8, Spin(8), triality, and an Alt(9) Schur double cover. Its role is representation-closure bookkeeping. It is not a proof of OPH, not a Standard Model selector, not a heterotic critical-worldsheet receipt, and not hardware evidence.

1 Role in the OPH Stack

OPH uses finite observer-visible data, records, repair maps, quotient invariants, and public receipts as primitive evidence carriers. The present certificate belongs to that finite exact discipline on the exceptional-symmetry side:

$$\text{Spin}(8)\text{ triality} + E_8\text{ lattice preservation} \quad \Rightarrow \quad \text{finite exceptional representation certificate.}$$

It supports the E_8-type representation-closure lane used by compact-gauge and heterotic-local bookkeeping. It does not replace the recovered-core theorem stack, the MAR selection of the Standard Model quotient, the critical-edge CFT gate, or any physical carrier evidence rule.

2 Certificate Statement

Theorem 1 (Finite E_8/Spin(8) triality certificate)**.** *The certificate target is an exact finite matrix construction with the following data.*

1. *An A_8 root subsystem inside the E_8 root lattice. Its Weyl group is $W(A_8) \cong \text{Sym}(9)$, and the even subgroup is* Alt(9).
2. *The permutation involution* $(12)(34) \in \text{Alt}(9)$ *lifts to* Spin(8) *with square* -1. *Hence the preimage of* Alt(9) *is the nonsplit Schur double cover* $2{\cdot}\text{Alt}(9)$, *not* $2 \times \text{Alt}(9)$.
3. *Under the positive half-spin representation Δ^+, the lifted group preserves an even unimodular determinant-one lattice, hence an E_8 lattice. The spin-side copy therefore lands in* $\text{Aut}(E_8) = W(E_8)$.

4. *The vector and positive-half-spin mod-2 orbit fingerprints on* $E_8/2E_8 \setminus \{0\}$ *are different:*

$$\mathrm{Alt}(9)_{\mathrm{vec}} : \{9, 36, 84, 126\}, \qquad (2\cdot\mathrm{Alt}(9))_{\Delta^+} : \{120, 135\}.$$

   Thus the two copies are not conjugate in $O_8^+(2)$*.*

5. *The outer triality automorphism of* Spin(8) *permutes the vector and two half-spin eight-dimensional representations and identifies these otherwise nonconjugate presentations.*

Remark 1 (Notation)*.* Here A_8 denotes the root subsystem and Alt(9) denotes the alternating group. The shorthand A_9 is avoided for the group because A_9 also denotes a root system.

Remark 2 (Claim boundary)*.* This is a finite algebraic certificate. It supports exceptional representation-closure and triality bookkeeping. It does not prove OPH, derive the Standard Model quotient, prove physical E_8 realization, close the heterotic edge CFT, or count as a hardware receipt.

Remark 3 (Public receipt gate)*.* For public verifier status, the repository bundle must include the Sage source, exact matrix data, lattice bases, mod-2 orbit computation, stdout or machine-readable check receipts, and stable hashes under `code/e8_triality/`. Until that bundle is populated, this note records the certificate statement and its OPH claim placement rather than a standalone public reproduction bundle.

3 Remaining Extension

This certificate concerns the nonsplit $2\cdot\mathrm{Alt}(9)$ subgroup and its triality-fused vector and positive-half-spin presentations. The full Griess–Lam 2.Sym(9) double-cover construction requires adjoining odd Clifford lifts. Those odd lifts exchange the two half-spin modules and carry the associated $\sqrt{2}$-normalization. That is a separate certificate gate.