The de Sitter Time-Advance Sign from a Finite Screen with Fixed Capacity

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Abstract

Negative de Sitter shocks produce a time advance and an increasing regularized out-of-time-ordered correlator. This sign conflicts with the positivity and cyclicity required by a proposed static-patch trace. Under a horizon-capacity dictionary, an exact fixed-budget law on a finite screen supplies a conditional sign mechanism. Maximizing generalized entropy over sector probabilities gives $S_{\mathrm{gen}}^{\mathrm{max}} = \log M$. Uniform transfer of a fraction f of the horizon capacity to an observer changes both this maximum and the uniformly normalized area observable by $\log(1-f) < 0$. The unperturbed horizon is therefore a one-sided boundary maximum along the transfer direction. The exact gradient and Hessian show that this statement cannot be replaced by an interior-maximum claim: at fixed horizon capacity the tangent Hessian is positive, while the homogeneous curvature is negative. For an icosahedral screen, a normalized finite shock spectrum follows conditionally from exact gauge transport of the rotation triplet and a nearest-neighbour Laplacian identification. The coefficient and gravitational attachment are open. The exact outputs are the pure-de-Sitter normalization, finite entropy maximum, transfer law, analytic curvature, and line-graph spectrum identity. The argument supplies no static-patch trace.

Keywords: de Sitter horizon, shock wave, static patch, finite screen, generalized entropy, icosahedral graph

1 The sign problem

Chen, Stanford, Tang, and Yang study a static-patch proposal in which a Euclidean gravitational path integral with an observer worldline computes a Hilbert-space trace [1]. Their shock-wave calculation includes observer recoil and gravitational backreaction. The resulting out-of-time-ordered correlator conflicts with cyclicity and positivity: its leading regularized contribution increases. Geometrically, the negative shock advances the signal.

The finite-screen calculation below isolates a capacity mechanism for this sign. Three levels remain separate throughout:

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1. exact entropy and graph identities for a finite sector model;
2. an analytic relaxation of integer sector dimensions for stating gradients and curvatures;
3. the physical dictionary identifying screen capacity transfer with a gravitational horizon shock.

The first two levels are mathematical statements. The third is a conditional attachment. In particular, a finite capacity premise does not evade the trace obstruction in Ref. [1]. Finite-dimensionality is part of the premise tested there.

2 The smooth de Sitter normalization

Write the horizon shock equation in spacetime dimension d as

$$\left(-\nabla^2_{S^{d-2}} - \mu^2\right) X^+ = 8\pi G\, T^-, \qquad \mu^2 = \frac{d-2}{2}|f'(r_c)|r_c = (d-2)\kappa r_c. \tag{1}$$

For pure de Sitter space of radius L,

$$f(r) = 1 - \frac{r^2}{L^2}, \qquad r_c = L, \qquad \kappa = \frac{1}{L}.$$

Lemma 2.1 (Radius-independent shock mass)**.** *For pure de Sitter space,*

$$\mu^2 = d - 2 = \lambda_{\ell=1}\left(S^{d-2}\right).$$

Consequently, μ^2 is independent of L and of the cosmological constant.

Proof. The product κr_c equals one. Equation (1) therefore gives $\mu^2 = d-2$. Scalar spherical harmonics on S^{d-2} have

$$\lambda_\ell = \ell(\ell + d - 3),$$

so $\lambda_1 = d - 2$. □

The smooth $\ell = 1$ shock is generated by a de Sitter isometry and is pure gauge, with the corresponding source component removed by the gravitational constraint [1]. This explains the zero of the operator in Eq. (1). Transferring that normalization to a discrete screen requires a separate premise, stated in Section 4.

3 Finite entropy and the capacity-transfer direction

3.1 The exact entropy maximum

Let a finite screen have positive integer sector dimensions $d_1, \ldots, d_n$, and let p_i be a probability distribution over the sectors. Set

$$M = \sum_{i=1}^n d_i, \qquad S_{\text{gen}}(p, d) = -\sum_{i=1}^n p_i \log p_i + \sum_{i=1}^n p_i \log d_i.$$

Proposition 3.1 (Finite sector entropy maximum)**.** *For fixed d_i,*

$$p_i^\star = \frac{d_i}{M}, \qquad \max_p S_{\mathrm{gen}}(p, d) = \log M.$$

Proof. With $q_i = d_i/M$,

$$S_{\mathrm{gen}}(p, d) = \log M - \sum_i p_i \log \frac{p_i}{q_i} = \log M - D_{\mathrm{KL}}(p\|q).$$

Nonnegativity of relative entropy proves the result. $\square$

Thus a logarithmic capacity $N = \log M_0$ agrees exactly with the extremal generalized entropy when the closed screen carries total dimension M_0. This is a maximization over p, with the sector dimensions held fixed. It does not classify extrema with respect to the d_i.

3.2 Exact gradient and Hessian

At the maximizing distribution, the expectation of the logarithmic sector operator is

$$\mathcal{A}(d) := \sum_{i=1}^{n} \frac{d_i}{M} \log d_i. \tag{2}$$

The dimensions are integers in the finite model. Extend them to positive real variables solely for the following differential calculation.

Proposition 3.2 (Analytic curvature of the sector observable)**.** *The exact gradient of Eq. (2) is*

$$\frac{\partial \mathcal{A}}{\partial d_i} = \frac{\log d_i + 1 - \mathcal{A}}{M}. \tag{3}$$

Writing $g_i = \partial_i \mathcal{A}$, its Hessian is

$$\frac{\partial^2 \mathcal{A}}{\partial d_i \partial d_j} = \frac{\delta_{ij}}{M d_i} - \frac{g_i + g_j}{M}. \tag{4}$$

At the symmetric point $d_i = d$,

$$\operatorname{Hess} \mathcal{A} = \frac{1}{nd^2}\left(I - \frac{2}{n}J\right), \tag{5}$$

where $J = \mathbf{1}\mathbf{1}^T$. Its eigenvalue on $\mathbf{1}$ is $-1/(nd^2)$, and every vector orthogonal to $\mathbf{1}$ has eigenvalue $+1/(nd^2)$.

Proof. Differentiate $\mathcal{A} = M^{-1}\sum_i d_i \log d_i$ to obtain Eq. (3). A second derivative gives Eq. (4). At $d_i = d$, one has $M = nd$, $\mathcal{A} = \log d$, and $g_i = 1/(nd)$, which yields Eq. (5). $\square$

Two consequences prevent a false extremum statement. First, $\nabla \mathcal{A} = \mathbf{1}/(nd)$ at the symmetric point, so the point is not stationary in the full positive orthant. Second, a fixed-horizon-capacity variation obeys $\mathbf{1}^T \delta d = 0$, and hence

$$\delta d^T (\operatorname{Hess} \mathcal{A}) \delta d = \frac{\|\delta d\|^2}{nd^2} > 0$$

for every nonzero tangent variation. The symmetric point is locally minimizing along the fixed-M redistribution directions. The negative eigenvalue belongs to the homogeneous direction, which changes M.

3.3 The one-sided budget maximum

The de Sitter sign concerns transfer between the horizon and the observer, rather than redistribution among horizon sectors. Let the closed total capacity be M_0. Assign $c_{\mathrm{obs}} \geq 0$ units in the finite capacity ledger to the observer, leaving

$$M_h = M_0 - c_{\mathrm{obs}}$$

on the horizon. Suppose the transfer depletes every horizon sector by the same factor $1 - f$, where $f = c_{\mathrm{obs}}/M_0$. For integer dimensions, restrict to values for which the depleted dimensions remain integers. Then

$$S^{\max}_{\mathrm{gen},h}(f) - S^{\max}_{\mathrm{gen},h}(0) = \log(1 - f), \tag{6}$$

and

$$\mathcal{A}((1 - f)d) - \mathcal{A}(d) = \log(1 - f). \tag{7}$$

Both relations are exact.

Theorem 3.3 (Finite transfer law and analytic one-sided maximum)**.** *Every admissible positive integer transfer has a smaller horizon entropy maximum and uniformly depleted area observable than the zero-transfer allocation. On the positive-real interpolation $0 \leq c_{\mathrm{obs}} < M_0$, the change is strictly decreasing and concave:*

$$\frac{d}{dc_{\mathrm{obs}}} \log(M_0 - c_{\mathrm{obs}}) = -\frac{1}{M_0 - c_{\mathrm{obs}}} < 0, \qquad \frac{d^2}{dc_{\mathrm{obs}}^2} \log(M_0 - c_{\mathrm{obs}}) = -\frac{1}{(M_0 - c_{\mathrm{obs}})^2} < 0.$$

The allocation $c_{\mathrm{obs}} = 0$ is a one-sided boundary maximum on both the admissible finite transfers and the analytic transfer path.

For transfers of 2%, 5%, and 10%,

$$\log(0.98) = -0.020203\ldots, \qquad \log(0.95) = -0.051293\ldots, \qquad \log(0.90) = -0.105361\ldots.$$

The exact finite order and its analytic interpolation are the budget statements relevant to the shock sign. Proposition 3.2 supplies compatible negative homogeneous curvature, while also showing why the transfer result cannot be described as an interior maximum at fixed horizon M.

3.4 Conditional gravitational interpretation

In the horizon coordinates used in Ref. [1], the constant term of the shock operator carries the sign of $B'(0)/B(0) = 2r'(0)/r_c$. For a cosmological horizon, the transverse area decreases from the horizon toward the observer, and the shock has the time-advance sign. Under the two physical identifications

$$\text{closed screen capacity} = \text{fixed horizon–observer budget}, \qquad \mathcal{A} = \frac{A_{\mathrm{horizon}}}{4G},$$

Theorem 3.3 supplies the corresponding conditional sign mechanism. Identifying the ledger coordinate c_{obs} with the observer mass used in the gravitational shock calculation is an additional physical dictionary. The finite entropy theorem and transfer law establish none of these identifications.

4 Conditional icosahedral spectrum

Consider the graph Laplacian of the twelve vertices of the icosahedron. Its eigenspaces decompose under A_5 as

$$\mathbb{R}^{12} \simeq \mathbf{1} \oplus \mathbf{3} \oplus \mathbf{5} \oplus \mathbf{3}',$$

with raw Laplacian eigenvalues

$$0, \qquad 5-\sqrt{5}, \qquad 6, \qquad 5+\sqrt{5},$$

respectively.

Assumption 4.1 (Discrete gauge transport DS-GAUGE)**.** *The* $\mathbf{3}$ *generated by the screen isometries consists of exact gauge zero modes of the discrete shock operator, in direct analogy with the smooth* $\ell = 1$ *isometry modes.*

Assumption 4.2 (Nearest-neighbour kinetic identification DS-LAPLACIAN)**.** *The discrete shock kinetic operator is the nearest-neighbour combinatorial Laplacian on the port graph, or its edge-sector line graph, with one positive overall scale.*

These two assumptions are independent. Assumption 4.1 fixes the scale by placing the $\mathbf{3}$ at the smooth value 2. Assumption 4.2 selects the finite operator whose remaining eigenvalues are being compared.

Proposition 4.3 (Conditional normalized shock spectrum)**.** *Under Assumptions 4.1 and 4.2, the normalized port Laplacian and shock spectra are*

A_5 *block*	*multiplicity*	λ_{norm}	λ_{shock}
$\mathbf{1}$	1	0	-2
$\mathbf{3}$	3	2	0
$\mathbf{5}$	5	$3+\dfrac{3}{\sqrt{5}}$	$1+\dfrac{3}{\sqrt{5}}$
$\mathbf{3}'$	3	$3+\sqrt{5}$	$1+\sqrt{5}$

The invariant mode agrees exactly with the smooth value -2*. The* $\mathbf{5}$ *shock eigenvalue is* $1+3/\sqrt{5} = 2.341640\ldots$*, and the additional* $\mathbf{3}'$ *triplet lies at* $1+\sqrt{5} = 2\varphi = 3.236067\ldots$*.*

Proof. Multiply the graph Laplacian by $2/(5-\sqrt{5})$, then subtract $\mu^2 = 2$. Direct simplification gives the displayed values. □

The value -2 carries no enhancement relative to the continuum invariant mode. Any different comparison obtained by leaving the graph scale unfixed is a normalization artifact.

4.1 Ports and edge sectors

The logarithmic sector operator naturally suggests an edge carrier. The low-lying spectrum is unchanged by this choice.

Lemma 4.4 (Regular line-graph spectrum)**.** *Let* G *be a* k*-regular graph with* $k \geq 2$*,* n *vertices, and* m *edges. The line graph* $L(G)$ *has the Laplacian spectrum of* G*, together with the eigenvalue* $2k$ *repeated* $m-n$ *times.*

Proof. Let B be the unoriented vertex-edge incidence matrix. Then

$$BB^T = A_G + kI, \qquad B^T B = A_{L(G)} + 2I.$$

The nonzero spectra of BB^T and $B^T B$ agree, and the spectrum of the $m \times m$ matrix $B^T B$ contains $m - n$ additional zeros relative to the spectrum of the $n \times n$ matrix BB^T, counted with multiplicity. Since $L(G)$ is $(2k-2)$-regular, converting adjacency eigenvalues to Laplacian eigenvalues gives the claim. □

For the icosahedron, $k = 5$, $n = 12$, and $m = 30$. The edge-sector spectrum is therefore

$$\{0^1, (5-\sqrt{5})^3, 6^5, (5+\sqrt{5})^3, 10^{18}\}.$$

It contains the full port spectrum plus a high block at 10. In particular, the normalization-free ratio between the **5** and rotation-triplet Laplacian eigenvalues is

$$\frac{\lambda_{\mathbf{5}}}{\lambda_{\mathbf{3}}} = \frac{6}{5-\sqrt{5}} = \frac{3(5+\sqrt{5})}{10} = 2.170820\ldots$$

on both carriers. The dodecahedral face graph gives $\varphi^2 = 2.618033\ldots$, while the smooth $\lambda_{\ell=2}/\lambda_{\ell=1}$ ratio is 3.

Lemma 4.4 closes the port-versus-edge ambiguity once the nearest-neighbour Laplacian premise is accepted. It does not establish that premise.

5 The repair generator has a different domain

Let X_r be the configuration space at fixed sector structure, with measure π_r, and let P_v be the single-site heat-bath conditional expectation. The repair generator

$$L^{\mathrm{rep}} = \sum_v (I - P_v)$$

acts on

$$K_r = L^2(X_r, \pi_r).$$

Since each P_v is an orthogonal projection,

$$\langle f, L^{\mathrm{rep}} f\rangle = \sum_v \|(I - P_v)f\|^2.$$

Consequently,

$$\ker L^{\mathrm{rep}} = \bigcap_v \operatorname{Ran} P_v.$$

Reducing this intersection to the constants requires a separate irreducibility or intersection proof and is unnecessary for the scope statement here.

A horizon shock changes the sector dimensions d_α in

$$L_C = \sum_\alpha (\log d_\alpha) P_\alpha.$$

Such a change deforms the configuration space and lies outside $L^2(X_r, \pi_r)$ at fixed sector structure. The repair generator therefore does not act on the sector-dimension shock. Its spectral gap neither excludes nor supplies the time-advance mode. The capacity-transfer law in Theorem 3.3 is a response input on the enlarged sector-dimension space; a dynamical generator on that space is open.

6 Claim boundary

Statement	Status
$\mu^2 = d - 2 = \lambda_{\ell=1}$ in pure de Sitter	exact smooth-geometry identity
$S_{\rm gen}^{\rm max} = \log M$	exact finite entropy maximum over sector probabilities
Uniform-depletion drop $\log(1-f)$	exact finite law for admissible integer dimensions
Gradient and Hessian of $\mathcal{A}(d)$	exact analytic-relaxation identity
One-sided horizon-budget maximum	exact order on admissible finite transfers and exact analytic interpolation; gravitational reading conditional
Port/edge low-spectrum equality	exact regular line-graph theorem
Normalized icosahedral shock spectrum	conditional on exact discrete gauge transport and the scaled nearest-neighbour Laplacian identification
Shock coefficient and physical screen attachment	open
Static-patch trace representation	excluded from the claim; the cited trace obstruction remains

The coefficient multiplying the shock response requires a generalized-entropy normalization and a physical horizon-area dictionary on one common domain. The finite calculation fixes neither. It also gives no Hilbert-space trace whose correlators satisfy the conditions tested in Ref. [1].

Closed tested classes and surviving alternatives

The external trace obstruction concerns the proposal tested in Ref. [1]: a Euclidean static-patch path integral with an observer worldline interpreted as a positive cyclic Hilbert-space trace for the stated correlators. Horizon-screen descriptions without that trace interpretation remain outside the tested class.

The repair-domain statement concerns the declared fixed-sector $L^{\rm rep}$ on $L^2(X_r, \pi_r)$. A separately constructed dimension-changing generator remains possible. The graph-spectrum statement concerns the scaled nearest-neighbour icosahedral port Laplacian and its line graph under exact gauge transport. Weighted, longer-range, refinement-limit, and other source-derived kinetic operators remain possible and require their own spectra.

None of these boundaries disproves Observer-Patch Holography. They delimit one trace interpretation, one fixed-sector generator, and one conditional finite kinetic operator. The exact finite entropy, transfer, and line-graph identities survive independently of those physical attachments.

7 Conclusion

A closed capacity budget has a distinguished transfer direction. Moving capacity from a horizon screen to its observer changes the extremal entropy and the uniformly normalized logarithmic area observable by $\log(1-f)$. The sign is negative for every allowed positive finite transfer and throughout the positive-real interpolation, so the zero-observer allocation is a one-sided boundary maximum. The analytic Hessian supports a negative homogeneous curvature and a positive fixed-horizon-capacity tangent curvature. Keeping both facts visible avoids an incorrect interior-maximum argument.

The pure de Sitter normalization $\mu^2 = \lambda_{\ell=1}$ is exact. An icosahedral spectrum follows only after exact discrete gauge transport and a nearest-neighbour kinetic operator are assumed. The regular line-graph identity then makes its low spectrum independent of choosing ports or edge sectors. These results supply a conditional finite-screen mechanism for the time-advance sign, with the ledger-to-mass map, coefficient, gravitational attachment, and sector-dimension dynamics left as separate obligations. The exact identities remain valid if those attachments fail; such a failure would reject the corresponding physical interpretation inside its stated class.

References

[1] Y. Chen, D. Stanford, H. Tang, and Z. Yang, *Negative shocks versus static patch holography*, arXiv:2607.14042 [hep-th], 2026. https://arxiv.org/abs/2607.14042