Theoretical Bounds on χ_ν in Observer-Patch Holography

Coherent-Matter Susceptibility and the Construction Window

Bernhard Mueller and Alex Osika

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Abstract

On the quotient-edge coherent-matter continuation, granting the dark-sector collar lemmas fixes χ_ν as a collar survival coefficient. The theorem-grade canonical band is

$$0.9343006394893864\ldots \leq \chi_\nu^{\text{can}} \leq 1,$$

and the exact uniform branch gives $\chi_\nu^{\text{can}} = e^{-P/24} = 0.9343006394893864\ldots$. Engineering estimates use a different chart. With a normalization factor of 10^{22}, the engineering number sits around 9.34×10^{-23} to 10^{-22}, with the exact branch at 9.34×10^{-23}. For a $100\,\text{kg}\,\text{m}^{-2}$ platform, full weight response asks for canonical coherence contrast around 8.6×10^{-9} to 9.2×10^{-9}. Ten percent assist asks for about 8.6×10^{-10} to 9.2×10^{-10}. The coefficient is in the useful mathematical range for hoverboard-class experiments. The engineering question is whether a real substrate can make that vertical scalar contrast in a controlled way.

Contents

1 Introduction

Keep three layers separate.

First, the recovered OPH core supplies the fixed-cutoff observer carrier, record algebras, quotient-local observables, the Einstein branch, and the canonical dark-sector scalar channel. That core leaves χ_ν unfixed. A zero coherent-matter continuation is allowed unless an additional continuation branch is declared.

Second, this paper declares such a branch. Coherent scalar sourcing is placed on the same quotient-edge cut register as the canonical dark-sector scalar repair opportunities. Granting the dark-sector collar lemmas on that shared register, the canonical susceptibility is the collar survival coefficient:

$$\chi_\nu^{\mathrm{can}} = \lambda_{\mathrm{collar}}.$$

The theorem belongs to the declared continuation branch. It is outside the recovered-core OPH tier.

Third, engineering calculations often use a different chart. They fold stored coherent energy into the scalar. That makes the coefficient look tiny, although the canonical coefficient is order one.

The canonical number is the OPH-invariant number. It uses the per-channel observer-facing scalar after the effective patch capacity has been divided out. On the exact uniform finite-thickness branch,

$$\chi_\nu^{\mathrm{can}} = e^{-P/24} = 0.9343006394893864\ldots.$$

With finite-thickness averaging and the protected reserve mean $P/24$, the theorem-grade band is

$$\boxed{0.9343006394893864 \le \chi_\nu^{\mathrm{can}} \le 1.}$$

The interval is narrow enough for engineering purposes. The exact branch is at the lower end of this interval.

The engineering chart number is

$$S_{\mathrm{coh}}^{\mathrm{eng}} = N_{\mathrm{coh}} S_{\mathrm{coh}}^{\mathrm{can}}, \qquad N_{\mathrm{coh}} = \varepsilon_{\mathrm{sub}} \frac{u_{\mathrm{stored}}}{u_0}.$$

The coefficient in that chart is smaller by the same factor:

$$\boxed{\chi_\nu^{\mathrm{eng}} = \frac{\chi_\nu^{\mathrm{can}}}{N_{\mathrm{coh}}}.}$$

This gives the engineering band

$$\boxed{\frac{0.9343006394893864}{N_{\mathrm{coh}}} \le \chi_\nu^{\mathrm{eng}} \le \frac{1}{N_{\mathrm{coh}}}.}$$

The translation is simple:

N_{coh}	χ_ν^{eng} range
10^{20}	9.34×10^{-21} to 1.00×10^{-20}
10^{21}	9.34×10^{-22} to 1.00×10^{-21}
10^{22}	9.34×10^{-23} to 1.00×10^{-22}
10^{23}	9.34×10^{-24} to 1.00×10^{-23}
10^{24}	9.34×10^{-25} to 1.00×10^{-24}

The engineering consequence is direct. The coefficient is large enough in the canonical chart. The hard part is the scalar: build a substrate with enough vertical canonical coherence contrast, keep ordinary ambient matter from creating the same scalar accidentally, and measure the force with clean controls. Values around 10^{-22} mean $N_{\mathrm{coh}} \sim 10^{22}$, while the canonical coefficient stays order one.

2 What the Bound Says for Devices

The branch theorem gives the canonical coefficient:

$$0.9343006394893864 \le \chi_\nu^{\mathrm{can}} \le 1.$$

On the exact uniform finite-thickness branch it is the single number

$$\chi_\nu^{\mathrm{can}} = 0.9343006394893864\ldots.$$

The susceptibility bound belongs to the declared continuation branch. It is an order-one coefficient. Zero is excluded on that branch.

A room-scale lift or weight-reduction device needs only tiny $\Delta\nu$. For areal load $\Sigma = M/A_\perp$ and target weight fraction f, the required response is

$$\Delta\nu_{\rm req} = f\,\frac{4\pi G\Sigma}{g} = 8.5525\times 10^{-9}\,f\,\Sigma_2, \qquad \Sigma_2 = \frac{\Sigma}{10^2\,{\rm kg\,m^{-2}}}.$$

The forced $\chi_\nu^{\rm can}$ band converts this into the required canonical coherence contrast:

$$\boxed{8.55\times 10^{-9}\,f\,\Sigma_2 \le \Delta S^{\rm can}_{\rm coh,req} \le 9.15\times 10^{-9}\,f\,\Sigma_2.}$$

For devices, the number to build is the coherence contrast. A $100\,{\rm kg\,m^{-2}}$ room-scale platform requires a vertical canonical coherence contrast of roughly 8.6×10^{-9} to 9.2×10^{-9} for full weight compensation. Ten percent compensation requires roughly 8.6×10^{-10} to 9.2×10^{-10}.

Device-scale case	Σ [kg m^{-2}]	f	$\Delta\nu_{\rm req}$	$\Delta S^{\rm can}_{\rm coh,req}$
light room platform	50	1	4.28×10^{-9}	4.28×10^{-9} to 4.58×10^{-9}
room-scale platform	100	1	8.55×10^{-9}	8.55×10^{-9} to 9.15×10^{-9}
heavy room platform	250	1	2.14×10^{-8}	2.14×10^{-8} to 2.29×10^{-8}
ten percent assist	100	0.1	8.55×10^{-10}	8.55×10^{-10} to 9.15×10^{-10}

Table 1: Room-scale response requirements implied by the branch-theorem canonical band. The table is independent of the engineering chart normalization.

For a hoverboard-class footprint the areal load is usually higher. A rider plus board spread over $0.5\,{\rm m^2}$ gives $\Sigma \simeq 200$ to $300\,{\rm kg\,m^{-2}}$. Full response then asks for

$$\Delta S^{\rm can}_{\rm coh,req} \simeq 1.7\times 10^{-8} \text{ to } 2.7\times 10^{-8}.$$

A compact footprint near $600\,{\rm kg\,m^{-2}}$ asks for about

$$5.1\times 10^{-8} \text{ to } 5.5\times 10^{-8}.$$

Ten percent assist divides these numbers by ten. The coefficient is in the useful range for hoverboard-class experiments. A practical hoverboard depends on whether a real substrate can produce and hold that vertical scalar contrast under load.

The engineering coefficient is a chart value:

$$\chi_\nu^{\rm eng} = \frac{\chi_\nu^{\rm can}}{N_{\rm coh}}.$$

The small engineering numbers are useful only together with the large engineering scalar. The response depends on the product

$$\chi_\nu^{\rm eng}\,\Delta S^{\rm eng}_{\rm coh} = \chi_\nu^{\rm can}\,\Delta S^{\rm can}_{\rm coh}.$$

For $N_{\rm coh} = 10^{22}$, the forced engineering band is

$$9.34\times 10^{-23} \le \chi_\nu^{\rm eng} \le 1.00\times 10^{-22}.$$

A $100\,\mathrm{kg\,m^{-2}}$ full-response platform then needs

$$\Delta S^{\rm eng}_{\rm coh,req} \simeq 8.6 \times 10^{13} \text{ to } 9.2 \times 10^{13}.$$

It equals 10^{22} times the canonical contrast in Table 1. The large scalar and small coefficient are the same normalization written in different coordinates.

On the declared branch, the coefficient is in the useful mathematical range for macroscopic weight-reduction devices. No room-temperature material claim follows from this bound alone. The engineering task is specific: generate a stable vertical $\Delta S^{\rm can}_{\rm coh}$ near 10^{-9} for assist and near 10^{-8} to 10^{-7} for full hoverboard-class response, keep ambient ordinary matter from creating the same scalar accidentally, and measure the force with controls that separate the effect from acoustic, thermal, electromagnetic, and mechanical couplings.

3 Status and Claim Boundary

The paper uses three claim tiers.

Tier A is the recovered OPH core: finite patch carriers, mismatch-lowering repair, observer-accessible record algebras, quotient normal forms, checkpoint continuation, support-visible BW scaling, fixed-cap generalized-entropy stationarity, and the Jacobson-type Einstein branch. These ingredients are supplied by the consensus, microphysics, synthesis, and compact SM/GR papers [3, 4, 1, 2]. This tier leaves χ_ν unfixed.

Tier B is the coherent-matter continuation law

$$\delta\nu = \chi^{\rm can}_\nu S^{\rm can}_{\rm coh} = \chi^{\rm eng}_\nu S^{\rm eng}_{\rm coh}. \tag{3.1}$$

This tier declares the response channel and the two charts without assigning a value for χ_ν.

Tier C is the conditional quotient-edge branch. It adds the dark-sector collar lemmas stated in Section 7. On this declared branch, coherent scalar opportunities are co-registered with the canonical dark-sector scalar repair channel, and $\chi^{\rm can}_\nu$ is forced to equal $\lambda_{\rm collar}$.

The dark-sector paper supplies the settled weak-field anomaly law

$$\rho_A = -\frac{1}{4\pi G}\nabla\cdot[(\nu_{\rm OPH}-1)\mathbf{g}_b], \tag{3.2}$$

interpreting the anomaly as a transported modular/collar information-defect remainder, with no additional particle species postulated [5].

Here $S^{\rm can}_{\rm coh}$ is the per-channel observer-facing scalar and $S^{\rm eng}_{\rm coh}$ is the stored-energy engineering scalar. The small values useful in device estimates are the engineering chart values $\chi^{\rm eng}_\nu = \chi^{\rm can}_\nu/N_{\rm coh}$.

4 Canonical Chain

4.1 Fixed-cutoff carrier

At fixed cutoff, an observer-facing OPH implementation surface is a federated patch carrier

$$\mathfrak{F} = (V, E, \{\mathcal{A}_i\}_{i\in V}, \{\mathcal{I}_e\}_{e\in E}, \{\pi_{i,e}\}, \{\mathcal{R}_i\}, \{\mathcal{U}_i\}). \tag{4.1}$$

Here $\mathcal{A}_i$ are local finite algebras, $\mathcal{I}_e$ are overlap interfaces, $\pi_{i,e}$ are visible restriction maps, $\mathcal{R}_i \subseteq Z(\mathcal{A}_i)$ are record algebras, and $\mathcal{U}_i$ are local update and repair interfaces. Physical claims are made through visible restrictions, record algebras, and quotient-local observables, without choosing hidden microscopic representatives [4].

4.2 Mismatch and repair

The consensus paper gives a nonnegative visible mismatch functional Φ. Accepted repairs lower the relevant mismatch on finite state spaces. Under the declared local-diamond and repair-completeness hypotheses, the terminal observer-facing normal form from a fixed initial physical quotient state is unique and independent of asynchronous update schedule [3].

4.3 Records and checkpoints

For an observer-supporting subfederation U, the microphysics paper uses a checkpoint of the form

$$\mathrm{Chk}_U(t) = \big(\mathcal{R}_U(t), \rho_U^{\mathrm{acc}}(t), \mathfrak{I}_U^{\mathrm{ext}}(t), \nu_{\geq t}, \mathfrak{B}_U(t)\big). \tag{4.2}$$

If two checkpoints agree on the accessible record algebra, accessible state, external interface, boundary-update schedule class, and provenance bundle, they induce the same continuation law on the observer-accessible event algebra. Approximate agreement gives a controlled total-variation bound [4].

4.4 Gravity and dark response

The compact SM/GR paper keeps the recovered core and downstream continuations separate. On the stated support-visible scaling branch, the OPH stack gives a Jacobson-type Einstein relation; dark-sector response laws are phenomenological continuations outside that recovered-core tier [2]. The dark paper then models the anomaly by Eq. (3.2).

On Earth, $g_b/a_0 \sim 10^{11}$, so the canonical ν_{OPH} term is pinned near unity. A terrestrial tabletop anomaly attributed only to the canonical OPH dark sector should read null. The χ_ν continuation is tested against that null baseline as a separate effect.

5 The Coherence Scalar

Definition 5.1 (Patch coherence factors and channel count)**.** *Let U be an observer-supporting patch subfederation at cycle t, with observer-accessible record algebra $\mathcal{Z}_U^{\mathrm{rec}}(t)$. Let $Y_U(t)$ be the induced record outcome variable, $X_{\partial U,t+1:t+h}$ the boundary packet over horizon h, and $\Phi_U(t)$ the visible mismatch on the relevant support. Define*

$$R_U(t;h) = [1 - d_{\mathrm{TV}}(\mathrm{Law}(Y_U(t+h)), \mathrm{Law}(Y_U(t)))]\, e^{-\delta_{\mathrm{rec}}(t)}, \tag{5.1}$$

$$P_U(t;h) = \frac{I(Y_U(t); X_{\partial U,t+1:t+h})}{H(X_{\partial U,t+1:t+h}) + \epsilon}, \tag{5.2}$$

$$C_U(t) = \left[1 - \frac{\Phi_U(t)}{\Phi_U^{\max} + \epsilon}\right]_+ . \tag{5.3}$$

On the screen or hardware branch, set the finite coherent channel count to

$$N_U(t) := \mathrm{Cap}(\mathcal{Z}_U^{\mathrm{rec}}(t)). \tag{5.4}$$

The exponent-free observer core is

$$\mathfrak{c}_U(t;h) = \mathbf{1}_{\text{self-read}}\, R_U(t;h)\, P_U(t;h)\, C_U(t). \tag{5.5}$$

Definition 5.2 (Canonical and engineering coherence scalars)**.** *The capacity-weighted observer scalar is*

$$\mathbb{C}_U(t;h) = N_U(t)\, \mathfrak{c}_U(t;h). \tag{5.6}$$

The canonical per-channel scalar is

$$\boxed{S_{\mathrm{coh}}^{\mathrm{can}}(U,t;h) = \frac{\mathbb{C}_U(t;h)}{N_U(t)} = \mathfrak{c}_U(t;h).} \tag{5.7}$$

On the quotient-edge continuation branch, $S_{\mathrm{coh}}^{\mathrm{can}} \in [0,1]$ is read as the fraction of coherent scalar opportunities on the edge-center cut register before the protected reserve acts. Let $u_{\mathrm{stored}}(U,t)$ be the stored coherent matter energy density in the substrate mode supported by U, let u_0 be a fixed reference energy density, and let $\varepsilon_{\mathrm{sub}}(U) \in [0,1]$ be the substrate directness factor. The engineering scalar is

$$S_{\mathrm{coh}}^{\mathrm{eng}}(U,t;h) = \varepsilon_{\mathrm{sub}}(U)\frac{u_{\mathrm{stored}}(U,t)}{u_0} S_{\mathrm{coh}}^{\mathrm{can}}(U,t;h) = N_{\mathrm{coh}}(U,t) S_{\mathrm{coh}}^{\mathrm{can}}(U,t;h), \tag{5.8}$$

where

$$\boxed{N_{\mathrm{coh}}(U,t) = \varepsilon_{\mathrm{sub}}(U)\frac{u_{\mathrm{stored}}(U,t)}{u_0}.} \tag{5.9}$$

Given a partition of unity $w_U(x)$ over observer-supporting subfederations, define the coarse-grained field

$$S_{\mathrm{coh}}^{\mathrm{can}}(x,t) = \sum_U w_U(x) S_{\mathrm{coh}}^{\mathrm{can}}(U,t;h), \qquad S_{\mathrm{coh}}^{\mathrm{eng}}(x,t) = \sum_U w_U(x) S_{\mathrm{coh}}^{\mathrm{eng}}(U,t;h), \qquad \sum_U w_U(x) = 1. \tag{5.10}$$

The continuation law can be written in either chart:

$$\boxed{\nu_{\mathrm{eff}}(x,t) = \nu_{\mathrm{OPH}}(x,t) + \chi_\nu^{\mathrm{can}} S_{\mathrm{coh}}^{\mathrm{can}}(x,t) = \nu_{\mathrm{OPH}}(x,t) + \chi_\nu^{\mathrm{eng}} S_{\mathrm{coh}}^{\mathrm{eng}}(x,t).} \tag{5.11}$$

On a region with representative N_{coh},

$$\boxed{\chi_\nu^{\mathrm{eng}} = \frac{\chi_\nu^{\mathrm{can}}}{N_{\mathrm{coh}}}.} \tag{5.12}$$

6 Structural Properties

Proposition 6.1 (Observer-facing quotient well-definedness)**.** *Assume two microscopic representatives induce the same observer-accessible continuation law on U, preserve the same visible mismatch score, and expose the same stored coherent energy density and substrate class. Then they give the same $S_{\mathrm{coh}}^{\mathrm{can}}(U,t;h)$ and $S_{\mathrm{coh}}^{\mathrm{eng}}(U,t;h)$.*

Proof. The factors R_U, P_U, and C_U are defined from observer-accessible records, boundary-law variables, and visible mismatch data. The channel count N_U cancels in the per-channel scalar. These are precisely the quantities that survive the observer-facing quotient. Multiplying by u_{stored} and ε_{sub}, which are declared observable substrate data for the engineering chart, preserves hidden-representative independence. □

Proposition 6.2 (Null conditions)**.** *If U has no self-read, no stable record algebra, or no predictive boundary coupling at horizon h, then $S^{\text{can}}_{\text{coh}}(U,t;h) = 0$ and $S^{\text{eng}}_{\text{coh}}(U,t;h) = 0$.*

Proof. If there is no self-read, then $\mathbf{1}_{\text{self-read}} = 0$. If no stable record algebra is available, then the stability factor R_U vanishes on the declared readout. If predictive boundary coupling is absent, then $P_U = 0$. In each case $\mathfrak{c}_U = 0$, hence both scalars vanish. □

Proposition 6.3 (Exact normalization map)**.** *For the canonical and engineering scalars in Eqs.* (5.7) *and* (5.8)*,*

$$\boxed{\frac{S^{\text{eng}}_{\text{coh}}(U,t;h)}{S^{\text{can}}_{\text{coh}}(U,t;h)} = N_{\text{coh}}(U,t) = \varepsilon_{\text{sub}}(U)\frac{u_{\text{stored}}(U,t)}{u_0}.} \tag{6.1}$$

Proof. Substitute Eq. (5.7) into Eq. (5.8). The quotient is exactly $\varepsilon_{\text{sub}} u_{\text{stored}}/u_0$. □

Proposition 6.4 (Saturated-limit reduction)**.** *If*

$$\mathbf{1}_{\text{self-read}} \simeq 1, \quad R_U \simeq 1, \quad P_U \simeq 1, \quad C_U \simeq 1,$$

then

$$S^{\text{can}}_{\text{coh}}(U,t;h) \simeq 1, \qquad S^{\text{eng}}_{\text{coh}}(U,t;h) \simeq \varepsilon_{\text{sub}}(U)\frac{u_{\text{stored}}(U,t)}{u_0}.$$

Proof. Substitute the saturated factor values into Eqs. (5.7) and (5.8). □

7 Canonical Quotient-Edge Susceptibility

Everything in this section is a conditional continuation statement. The recovered OPH core supplies the observer-facing carrier and the scalar channel. The nonzero coefficient follows only after the following dark-sector branch lemmas are granted.

Assumption 7.1 (Granted dark-sector collar lemmas)**.** *L1. Positive scalar source branch: the scalar repair defect is the minimal recoverability defect $R_C = I(A : D \mid B) \geq 0$.*

L2. Quotient-edge scalar locality: every quotient-local scalar recoverability event is resolved on the edge-center cut register.

L3. Co-registration with the protected reserve: scalar activation and the protected $\mathbb{Z}_6$ reserve live on the same edge-center resolution.

L4. Local Poisson reserve survival: on a co-registered scalar slot at transverse coordinate y,

$$\lambda_{\text{slot}}(y) = \exp[-\epsilon_{\mathbb{Z}_6}(y)].$$

L5. Finite-thickness averaging:

$$\lambda_{\mathrm{collar}} = \int \mathrm{d}y\, w(y) \exp[-\epsilon_{\mathbb{Z}_6}(y)], \qquad \int \mathrm{d}y\, w(y)\epsilon_{\mathbb{Z}_6}(y) = \frac{P}{24}.$$

L6. Exact uniform branch: on the product transverse regulator with one quotient trace and uniform protected-center density,

$$\lambda_{\mathrm{collar}} = e^{-P/24}.$$

The public OPH readout gives

$$P = 1.630968209403959, \qquad \frac{P}{24} = 0.06795700872516496. \tag{7.1}$$

Theorem 7.2 (Forced canonical susceptibility on the co-registered branch)**.** *Assume L1 to L5 of Assumption 7.1. Then the canonically normalized coherent-matter susceptibility is the collar survival coefficient:*

$$\boxed{\chi_\nu^{\mathrm{can}} = \lambda_{\mathrm{collar}}.} \tag{7.2}$$

Proof. By definition, $S^{\mathrm{can}}_{\mathrm{coh}}$ is the fraction of coherent scalar opportunities present on the quotient-edge cut register before the protected reserve acts. By L3, the $\mathbb{Z}_6$ reserve acts on that same register. By L4, each local coherent scalar opportunity survives the reserve with factor $\exp[-\epsilon_{\mathbb{Z}_6}(y)]$. By L5, finite-thickness averaging gives the surviving coherent scalar fraction

$$S^{\mathrm{can}}_{\mathrm{coh,after}} = \lambda_{\mathrm{collar}} S^{\mathrm{can}}_{\mathrm{coh,before}}.$$

The continuation law writes the same surviving fraction as

$$S^{\mathrm{can}}_{\mathrm{coh,after}} = \chi_\nu^{\mathrm{can}} S^{\mathrm{can}}_{\mathrm{coh,before}}.$$

For any nonzero canonical coherent fraction, the coefficients are equal. □

Corollary 7.3 (Theorem-grade nonzero band)**.** *Under L1 to L5,*

$$\boxed{0.9343006394893864\ldots \leq \chi_\nu^{\mathrm{can}} \leq 1.} \tag{7.3}$$

Proof. The function e^{-x} is convex, so the Jensen inequality gives

$$\chi_\nu^{\mathrm{can}} = \int \mathrm{d}y\, w(y) e^{-\epsilon_{\mathbb{Z}_6}(y)} \geq \exp\Big[-\int \mathrm{d}y\, w(y)\epsilon_{\mathbb{Z}_6}(y)\Big] = e^{-P/24}.$$

Using Eq. (7.1),

$$e^{-P/24} = 0.9343006394893864\ldots.$$

Reserve survival factors are at most one, so $\chi_\nu^{\mathrm{can}} \leq 1$. □

Corollary 7.4 (Exact canonical value on the uniform branch)**.** *Assume L6 in addition to L1 to L5. Then*

$$\boxed{\chi_\nu^{\mathrm{can}} = e^{-P/24} = 0.9343006394893864\ldots.} \tag{7.4}$$

Proof. By L6, $\lambda_{\mathrm{collar}} = e^{-P/24}$. Use Eq. (7.2). □

Remark 7.5 (Profile-envelope number). The dark-sector correction audit also lists profile targets: best same-function RAR 0.958802256, binned RAR 0.936737944, and the OPH $\mathbb{Z}_6$/Poisson value 0.934300639. If an extra numerical selector keeps the coherent continuation inside that same-function profile envelope, one may quote the practical profile range

$$0.934300639 \le \chi_\nu^{\text{can}} \le 0.958802256.$$

The range is selector-dependent. The theorem-grade band used in the rest of this paper is Eq. (7.3), with the exact value Eq. (7.4) on L6.

Corollary 7.6 (Engineering chart band)**.** *On any engineering chart with representative normalization N_{coh},*

$$\boxed{\frac{0.93430063894893864}{N_{\text{coh}}} \le \chi_\nu^{\text{eng}} \le \frac{1}{N_{\text{coh}}}.} \tag{7.5}$$

On the exact uniform branch,

$$\boxed{\chi_\nu^{\text{eng}} = \frac{e^{-P/24}}{N_{\text{coh}}}.} \tag{7.6}$$

N_{coh}	theorem-grade χ_ν^{eng} band	exact-branch χ_ν^{eng}
10^{20}	9.34×10^{-21} to 1.00×10^{-20}	9.34×10^{-21}
10^{21}	9.34×10^{-22} to 1.00×10^{-21}	9.34×10^{-22}
10^{22}	9.34×10^{-23} to 1.00×10^{-22}	9.34×10^{-23}
10^{23}	9.34×10^{-24} to 1.00×10^{-23}	9.34×10^{-24}
10^{24}	9.34×10^{-25} to 1.00×10^{-24}	9.34×10^{-25}

Table 2: Translation from the theorem-grade canonical band to the engineering chart. Here $N_{\text{coh}} = \varepsilon_{\text{sub}} u_{\text{stored}}/u_0$.

8 Weak-Field Response

Insert Eq. (5.11) into the settled weak-field density:

$$\rho_A^{\text{eff}} = -\frac{1}{4\pi G}\nabla \cdot \Big[\big(\nu_{\text{OPH}} - 1 + \chi_\nu^{\text{eng}} S_{\text{coh}}^{\text{eng}}\big)\mathbf{g}_b\Big]. \tag{8.1}$$

In a small terrestrial laboratory region with $\mathbf{g}_b \simeq -g\,\hat{z}$ and canonical $\nu_{\text{OPH}} - 1$ negligible, this becomes

$$\rho_A^{\text{lab}} \simeq -\frac{1}{4\pi G}\nabla \cdot [\chi_\nu^{\text{eng}} S_{\text{coh}}^{\text{eng}}\, \mathbf{g}]. \tag{8.2}$$

Theorem 8.1 (Planar response law)**.** *Let a thin device have horizontal projected area $A_\perp$, and define the bottom-to-top coherence contrast*

$$\Delta S_{\text{coh}}^{\text{eng}} = S_{\text{coh}\,\text{bottom}}^{\text{eng}} - S_{\text{coh}\,\text{top}}^{\text{eng}}.$$

Then, to leading order in the thin-device approximation, the apparent vertical force generated by the engineering-chart continuation is

$$\boxed{F_\chi = \frac{g^2}{4\pi G} A_\perp\, \chi_\nu^{\text{eng}}\, \Delta S_{\text{coh}}^{\text{eng}}.} \tag{8.3}$$

Proof. With $\mathbf{g} = -g\hat{z}$, Eq. (8.2) gives

$$\rho_A^{\rm lab} = \frac{g\chi_\nu^{\rm eng}}{4\pi G}\frac{\partial S_{\rm coh}^{\rm eng}}{\partial z}$$

up to the sign convention for z. Integrating through a thin column gives

$$M_A = \int \rho_A^{\rm lab}\,{\rm d}V = -\frac{g\chi_\nu^{\rm eng}}{4\pi G}A_\perp\,\Delta S_{\rm coh}^{\rm eng}$$

when $\Delta S_{\rm coh}^{\rm eng}$ is bottom minus top. The apparent weight change is $F_\chi = -gM_A$, yielding Eq. (8.3). □

Corollary 8.2 (Geometric ceiling)**.** *If* $|\Delta\nu| \lesssim 1$, *then*

$$\frac{|F_\chi|}{A_\perp} \lesssim C_g \equiv \frac{g^2}{4\pi G} = 1.1466 \times 10^{11}\,{\rm N\,m^{-2}} \tag{8.4}$$

for $g = 9.80665\,{\rm m\,s^{-2}}$ *and* $G = 6.67430 \times 10^{-11}\,{\rm m^3\,kg^{-1}\,s^{-2}}$.

The ceiling is algebraic. Material reachability of $|\Delta\nu| = 1$ is a separate claim.

9 Engineering Susceptibility Window

For bounds it is useful to package the contrast geometry, substrate directness, and OPH coherence factors into one dimensionless number:

$$\Delta S_{\rm coh}^{\rm eng} = \Gamma_{\rm eff}\frac{u}{u_0}, \qquad 0 \le \Gamma_{\rm eff} \le 1. \tag{9.1}$$

Here u is the representative stored coherent energy density in the active mode. The factor $\Gamma_{\rm eff}$ includes vertical contrast, substrate directness, and effective canonical observer coherence.

Theorem 9.1 (Nonempty macroscopic linear-response window)**.** *Let* $\Sigma = M/A_\perp$ *be the areal load, let* $f \in [0, 1]$ *be the target fraction of the ordinary weight* Mg, *and let* $\epsilon_{\rm lin} \le 1$ *be the maximum allowed* $|\Delta\nu|$ *in the chosen linear-response regime. Assume* $\Gamma_{\rm eff} > 0$. *If*

$$f\,\frac{4\pi G\Sigma}{g} < \epsilon_{\rm lin}, \tag{9.2}$$

then the interval

$$\boxed{\frac{4\pi G\Sigma f}{g}\,\frac{u_0}{\Gamma_{\rm eff}u} \le |\chi_\nu^{\rm eng}| \le \epsilon_{\rm lin}\frac{u_0}{\Gamma_{\rm eff}u}} \tag{9.3}$$

is nonempty. Values in this interval produce at least fraction f of the ordinary weight inside the declared linear-response bound. Conversely, any value that produces at least fraction f while staying inside that linear-response bound lies in this interval.

Proof. The force law gives

$$\frac{|F_\chi|}{Mg} = \frac{g}{4\pi G\Sigma}|\chi_\nu^{\rm eng}|\Delta S_{\rm coh}^{\rm eng}.$$

Requiring $|F_\chi|/(Mg) \ge f$ gives the lower bound in Eq. (9.3). Requiring $|\Delta\nu| = |\chi_\nu^{\rm eng}|\Delta S_{\rm coh}^{\rm eng} \le \epsilon_{\rm lin}$ gives the upper bound. The interval is nonempty exactly when the lower bound is smaller than the upper bound, which is Eq. (9.2). □

Corollary 9.2 (Numerical width in normalized areal load)**.** *Let*

$$\Sigma_2 = \frac{\Sigma}{10^2\,\mathrm{kg\,m^{-2}}}.$$

Then

$$\frac{4\pi G\Sigma}{g} = 8.5525\times 10^{-9}\,\Sigma_2.$$

The full-response $(f=1)$*, order-unity linear window has width*

$$\frac{\chi_{\rm upper}}{\chi_{\rm lower}} = 1.1692\times 10^{8}\,\Sigma_2^{-1},$$

independent of u *and* $\Gamma_{\rm eff}$*. With* $\epsilon_{\rm lin}=0.1$*, the window width is* $1.1692\times 10^{7}\,\Sigma_2^{-1}$.

$u\,[\mathrm{J\,m^{-3}}]$	u/u_0	$\lvert\chi_\nu^{\rm eng}\rvert_{\rm req}$ for $f\Sigma_2/\Gamma_{\rm eff}=1$	$\lvert\chi_\nu^{\rm eng}\rvert_{\rm lin}$ for $\epsilon_{\rm lin}/\Gamma_{\rm eff}=1$
1.0×10^{5}	1.18×10^{14}	7.27×10^{-23}	8.5×10^{-15}
5.5×10^{5}	6.47×10^{14}	1.32×10^{-23}	1.55×10^{-15}
1.0×10^{6}	1.18×10^{15}	7.27×10^{-24}	8.5×10^{-16}
4.0×10^{6}	4.71×10^{15}	1.82×10^{-24}	2.13×10^{-16}
1.0×10^{7}	1.18×10^{16}	7.27×10^{-25}	8.5×10^{-17}

Table 3: Full-response susceptibility bounds for $u_0 = 8.5\times10^{-10}\,\mathrm{J\,m^{-3}}$ and $\Sigma_2 = \Sigma/(10^2\,\mathrm{kg\,m^{-2}})$. The third column should be multiplied by $f\Sigma_2/\Gamma_{\rm eff}$. The fourth column should be multiplied by $\epsilon_{\rm lin}/\Gamma_{\rm eff}$.

The table gives the bound in numerical form. It says that ordinary material stress and coherent phonon energy densities put the macroscopic response threshold in the $\chi_\nu^{\rm eng}\sim10^{-22}$ to 10^{-24} range for macroscopic areal loads and $\Gamma_{\rm eff}$ near unity. Smaller $\Gamma_{\rm eff}$ shifts the required value upward by $1/\Gamma_{\rm eff}$.

10 Power and Stored-Energy Bounds

The force law fixes stored energy. Input power depends on how the coherent state is maintained.

10.1 Driven coherent oscillations

For a driven oscillator with angular frequency ω, mode volume $V_{\rm mode}$, quality factor Q, and stored density u,

$$P_{\rm loss} = \frac{uV_{\rm mode}\omega}{Q}. \tag{10.1}$$

Combining Eq. (10.1) with the required density from the macroscopic response law gives

$$\boxed{P_{\rm loss} = \frac{4\pi G\Sigma f\,u_0\,V_{\rm mode}\omega}{g\,Q\,\Gamma_{\rm eff}\,\lvert\chi_\nu^{\rm eng}\rvert}}. \tag{10.2}$$

For $\Sigma_2 = 1$, $V_{\text{mode}} = 10^{-3}\,\text{m}^3$, $\omega = 2\pi(4.0 \times 10^4)\,\text{s}^{-1}$, $Q = 10^5$, $f = 1$, and $\Gamma_{\text{eff}} = 1$, this gives:

$\lvert\chi_\nu^{\text{eng}}\rvert$	P_{loss}
10^{-20}	1.8 W
10^{-21}	18 W
10^{-22}	183 W
10^{-23}	1.8 kW
10^{-24}	18 kW

The scaling is $P_{\text{loss}} \propto (Q\Gamma_{\text{eff}}\lvert\chi_\nu^{\text{eng}}\rvert)^{-1}$.

10.2 Static elastic storage

For a static elastic-strain coherent variable, the stored density is

$$u_{\text{elastic}} = \frac{\sigma^2}{2E}, \tag{10.3}$$

where σ is stress and E is the Young modulus. In that interpretation there is no oscillator decay factor ω/Q. Maintaining the state is then limited by creep, relaxation, and mechanical failure. Its loss law differs from Eq. (10.1). The same susceptibility bounds apply, because they depend only on u, Γ_{eff}, and Σ.

11 Construction-Scale Consequences

The bounds are algebraic. Substrate reachability of $\Gamma_{\text{eff}} \simeq 1$ and safe maintenance of a high-energy coherent state are separate engineering claims. The bounds specify the requirements for a macroscopic force device if the continuation is real.

A force device, in this narrow sense, is a bounded substrate whose active region creates a vertical engineering-coherence contrast $\Delta S_{\text{coh}}^{\text{eng}}$. The apparent weight fraction is

$$\frac{\lvert F_\chi\rvert}{Mg} = \frac{g}{4\pi G\Sigma}\,\lvert\chi_\nu^{\text{eng}}\rvert\,\Gamma_{\text{eff}}\,\frac{u}{u_0}. \tag{11.1}$$

All construction choices enter through the product

$$\lvert\chi_\nu^{\text{eng}}\rvert\,\Gamma_{\text{eff}}\,u. \tag{11.2}$$

The product makes χ_ν^{eng} the governing chart number. Geometry fixes the coefficient $g/(4\pi G\Sigma)$. Better materials can raise u. Better mode design can raise Γ_{eff}. A smaller χ_ν^{eng} must be paid for linearly by higher stored energy density, larger active volume, better quality factor, or a smaller target response fraction. If $\chi_\nu^{\text{eng}} = 0$, no amount of construction produces the continuation force.

For the normalized areal load $\Sigma_2 = 1$, the full-response condition is

$$\Delta\nu_{\text{full}}(\Sigma_2 = 1) = \frac{4\pi G\Sigma}{g} = 8.5525 \times 10^{-9}. \tag{11.3}$$

$\lvert\chi_\nu^{\rm eng}\rvert$	$u_{\rm req}$ at $f\Sigma_2/\Gamma_{\rm eff}=1$	$P_{\rm loss}$ at $f\Sigma_2/\Gamma_{\rm eff}=1, Q=10^5$	Construction class	Interpretation
10^{-20}	$7.3\times10^{2}\,\mathrm{J\,m^{-3}}$	$1.8\,\mathrm{W}$	small bench article	The force is easy to produce if the coherence scalar is real. Existing ambient-coherence nulls become the primary consistency check.
10^{-21}	$7.3\times10^{3}\,\mathrm{J\,m^{-3}}$	$18\,\mathrm{W}$	tabletop module	Accessible to modest driven resonators or static strain specimens. A direct balance or pendulum experiment should see it cleanly.
10^{-22}	$7.3\times10^{4}\,\mathrm{J\,m^{-3}}$	$183\,\mathrm{W}$	high-Q insert / compact platform	The first useful construction band. Full response is compatible with ordinary high-Q crystal or metal inserts. Thermal management matters.
10^{-23}	$7.3\times10^{5}\,\mathrm{J\,m^{-3}}$	$1.8\,\mathrm{kW}$	engineered crystal or metal module	Inside high-strength material energy densities. Continuous driven operation becomes hard. Static elastic storage becomes much more attractive.
10^{-24}	$7.3\times10^{6}\,\mathrm{J\,m^{-3}}$	$18\,\mathrm{kW}$	industrial multi-module system	Near the upper end of practical stored-density claims. Driven operation needs large area, high Q, cryogenic or exceptional materials, or partial-lift use.
10^{-25}	$7.3\times10^{7}\,\mathrm{J\,m^{-3}}$	$183\,\mathrm{kW}$	large array / static-only candidate	Inside the response window. Compact driven construction is implausible. Static prestress, very high $\Gamma_{\rm eff}$, or large arrays are the plausible routes.
10^{-26}	$7.3\times10^{8}\,\mathrm{J\,m^{-3}}$	$1.8\,\mathrm{MW}$	outside compact construction	The response inequalities are satisfied. Material and power requirements exceed the intended compact-device scale. For compact devices, this row is effectively a null.

Table 4: Construction regimes inside the response-window theorem for $\Sigma_2 = 1$, $f = 1$, $u_0 = 8.5\times10^{-10}\,\mathrm{J\,m^{-3}}$, $V_{\rm mode} = 10^{-3}\,\mathrm{m^3}$, $\omega = 2\pi(4.0\times10^4)\,\mathrm{s^{-1}}$, and $Q = 10^5$. Every row has $\Delta\nu = 8.5525\times10^{-9}$, so construction difficulty changes while algebraic admissibility is the same. Multiply the listed material and power requirements by $f\Sigma_2/\Gamma_{\rm eff}$.

The value is far below even a conservative $\epsilon_{\text{lin}} = 0.1$ linear bound. The required $\Delta\nu$ lies inside the theorem window. Construction depends on the required $\Gamma_{\text{eff}} u$ product, mode volume, quality factor, and material stress.

Two consequences follow.

First, a measurement of χ_ν^{eng} selects a construction class. If $|\chi_\nu^{\text{eng}}| \gtrsim 10^{-22}$ and Γ_{eff} is reasonably large, compact demonstrators are in the ordinary resonator and high-strength-substrate regime. If $|\chi_\nu^{\text{eng}}| \sim 10^{-23}$ to 10^{-24}, the design must move toward high-Q crystals, larger active area, static strain, or partial lift. If $|\chi_\nu^{\text{eng}}| \lesssim 10^{-25}$, compact force-device construction stops being the right first target. The useful output is a bound on the continuation.

Second, the static-versus-driven distinction is decisive. The driven column pays $uV_{\text{mode}}\omega/Q$. Static elastic storage pays no oscillator maintenance cost. It must survive creep and fracture. The same χ_ν^{eng} value can be impractical for a driven phonon design and practical for a static strain design. The static-stress experiment decides which maintenance law applies to the construction table.

12 Experimental Bound Formula

Let an experiment have force sensitivity $F_{\min}$, projected area $A_\perp$, and known engineering-coherence contrast $\Delta S_{\text{coh}}^{\text{eng}}$. A null result gives

$$\boxed{|\chi_\nu^{\text{eng}}| \le \frac{4\pi G F_{\min}}{g^2 A_\perp |\Delta S_{\text{coh}}^{\text{eng}}|}.} \tag{12.1}$$

If the mode is a driven oscillator, with $\Delta S_{\text{coh}}^{\text{eng}} = \Gamma_{\text{eff}} Q P_{\text{in}}/(u_0 \omega V_{\text{mode}})$, then

$$\boxed{|\chi_\nu^{\text{eng}}| \le \frac{4\pi G F_{\min} u_0 \omega V_{\text{mode}}}{g^2 A_\perp \Gamma_{\text{eff}} Q P_{\text{in}}}.} \tag{12.2}$$

If the mode is static elastic storage, replace the driven density by u_{elastic} from Eq. (10.3).

13 Compatibility with Existing Nulls

Proposition 13.1 (No ordinary-matter anomaly from null coherence)**.** *If ordinary non-engineered configurations have $\mathfrak{c}_U = 0$ on the observer-facing scalar of Eq. (5.5), then they impose no direct bound on χ_ν^{eng}, because $\delta\nu = \chi_\nu^{\text{eng}} S_{\text{coh}}^{\text{eng}} = 0$ for those configurations.*

Proof. Equation (5.8) gives the statement immediately. The continuation couples only to the OPH-native coherent scalar. Undifferentiated rest mass or heat is outside this ansatz. □

If ordinary matter has a small nonzero ambient scalar $S_{\text{coh amb}}^{\text{eng}}$, then existing gravitational null tests bound the product:

$$|\chi_\nu^{\text{eng}}|\, S_{\text{coh amb}}^{\text{eng}} \le \delta\nu_{\text{test}}. \tag{13.1}$$

The condition is real. It becomes a theorem-grade number when the ambient scalar is specified. The continuation is falsifiable in two ways: by direct χ_ν^{eng} searches through Eq. (12.1) and by ordinary-gravity null tests once a model for $S_{\text{coh amb}}^{\text{eng}}$ is fixed.

14 How to Read the Two Bands

The canonical band in Eq. (7.3) is the quotient-edge continuation statement. It is dimensionless and independent of material energy density. It belongs to the branch where coherent scalar sourcing is co-registered with the dark-sector scalar repair channel. On the exact uniform-reserve branch, the value is the single number $0.9343006394893864\ldots$.

The engineering band in Eq. (7.5) is the same statement in a stored-energy chart. It depends on the declared normalization $N_{\text{coh}} = \varepsilon_{\text{sub}} u_{\text{stored}}/u_0$. Large coherent stored energy densities make χ_ν^{eng} numerically small because the scalar itself has been scaled up.

The response window in Eq. (9.3) is an operating condition for a chosen device geometry and linear-response allowance. A design with fixed Σ, f, u, and Γ_{eff} is compatible with the OPH continuation coefficient when the forced engineering band intersects that operating window. If the forced band sits above the chosen linear ceiling, the design is outside that linear chart. If it sits below the target threshold, the target response is too large for that design.

15 Chart Choices

The canonical scalar uses the exponent-free observer core $\mathbf{1}_{\text{self-read}} R_U P_U C_U$. Optional empirical exponents or refinement factors are calibrated-chart data. They are outside the canonical quotient-edge coefficient.

The coarse-grained partition weights $w_U(x)$ should be induced by a declared observer-supporting subcover. Only the contrast entering the force law is used in the response bounds.

The exact value $e^{-P/24}$ uses the uniform-reserve finite-thickness branch. The wider numerical interval in Eq. (7.3) uses finite-thickness averaging and the protected reserve mean. The profile-envelope range in the remark after the exact-value corollary needs an extra numerical selector.

16 Conclusion

The recovered OPH core leaves χ_ν unfixed. This paper defines a coherent-matter continuation branch with a canonical quotient-edge chart and an engineering stored-energy chart:

$$\nu_{\text{eff}} = \nu_{\text{OPH}} + \chi_\nu^{\text{can}} S_{\text{coh}}^{\text{can}} = \nu_{\text{OPH}} + \chi_\nu^{\text{eng}} S_{\text{coh}}^{\text{eng}}.$$

On the quotient-edge co-registered branch, granting the dark-sector collar lemmas forces the canonical coefficient to equal the collar survival coefficient:

$$\chi_\nu^{\text{can}} = \lambda_{\text{collar}}.$$

L1 to L5 give the theorem-grade band

$$0.9343006394893864\ldots \leq \chi_\nu^{\text{can}} \leq 1.$$

L6 gives the exact value

$$\chi_\nu^{\text{can}} = e^{-P/24} = 0.9343006394893864\ldots.$$

This excludes $\chi_\nu^{\mathrm{can}} = 0$ on the declared quotient-edge continuation.

In the engineering chart,

$$\chi_\nu^{\mathrm{eng}} = \frac{\chi_\nu^{\mathrm{can}}}{N_{\mathrm{coh}}}, \qquad N_{\mathrm{coh}} = \varepsilon_{\mathrm{sub}} \frac{u_{\mathrm{stored}}}{u_0}.$$

This gives

$$\frac{0.9343006394893864}{N_{\mathrm{coh}}} \le \chi_\nu^{\mathrm{eng}} \le \frac{1}{N_{\mathrm{coh}}}.$$

Engineering values near 10^{-22} correspond to $N_{\mathrm{coh}} \sim 10^{22}$. The response-window inequalities then decide which device geometries and material states stay inside the chosen linear-response chart.

For $\Sigma = 100\,\mathrm{kg\,m^{-2}}$, full weight response requires

$$\Delta S_{\mathrm{coh,req}}^{\mathrm{can}} \simeq 8.6 \times 10^{-9} \text{ to } 9.2 \times 10^{-9},$$

and ten percent assist requires about 8.6×10^{-10} to 9.2×10^{-10}. A hoverboard-class areal load around 200 to $600\,\mathrm{kg\,m^{-2}}$ moves the full-response target to about 2×10^{-8} to 6×10^{-8}. The coefficient is compatible with those room-scale response levels on the declared branch. The engineering task is the construction of a substrate that produces the required vertical canonical contrast with stable controls and acceptable losses.

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